

Saber Elaydi

An Introduction to Difference Equations

Third Edition

 Springer

Saber Elaydi
Department of Mathematics
Trinity University
San Antonio, Texas 78212
USA

Editorial Board

S. Axler
Mathematics Department
San Francisco State
University
San Francisco, CA 94132
USA

F.W. Gehring
Mathematics Department
East Hall
University of Michigan
Ann Arbor, MI 48109
USA

K.A. Ribet
Department of
Mathematics
University of California
at Berkeley
Berkeley, CA 94720-3840
USA

Mathematics Subject Classification (2000): 12031

Library of Congress Cataloging-in-Publication Data
Elaydi, Saber, 1943–

An introduction to difference equations / Saber Elaydi. — 3rd ed.

p. cm. — (Undergraduate texts in mathematics)

Includes bibliographical references and index.

ISBN 0-387-23059-9 (acid-free paper)

1. Difference equations. I. Title. II. Series.

QA431.E43 2005

515'.625—dc22

2004058916

ISBN 0-387-23059-9

Printed on acid-free paper.

© 2005 Springer Science+Business Media, Inc.

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer Science+Business Media, Inc., 233 Spring Street, New York, NY 10013, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use in this publication of trade names, trademarks, service marks, and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

Printed in the United States of America.

(MV)

9 8 7 6 5 4 3 2 1

SPIN 10950678

springeronline.com

Contents

Preface to the Third Edition	v
Preface to the Second Edition	ix
Preface to the First Edition	xi
List of Symbols	xx
1 Dynamics of First-Order Difference Equations	1
1.1 Introduction	1
1.2 Linear First-Order Difference Equations	2
1.2.1 Important Special Cases	4
1.3 Equilibrium Points	9
1.3.1 The Stair Step (Cobweb) Diagrams	13
1.3.2 The Cobweb Theorem of Economics	17
1.4 Numerical Solutions of Differential Equations	20
1.4.1 Euler's Method	20
1.4.2 A Nonstandard Scheme	24
1.5 Criterion for the Asymptotic Stability of Equilibrium Points	27
1.6 Periodic Points and Cycles	35
1.7 The Logistic Equation and Bifurcation	43
1.7.1 Equilibrium Points	43
1.7.2 2-Cycles	45
	xv

1.7.3	2 ² -Cycles	46
1.7.4	The Bifurcation Diagram	47
1.8	Basin of Attraction and Global Stability (Optional)	50
2	Linear Difference Equations of Higher Order	57
2.1	Difference Calculus	57
2.1.1	The Power Shift	59
2.1.2	Factorial Polynomials	60
2.1.3	The Antidifference Operator	61
2.2	General Theory of Linear Difference Equations	64
2.3	Linear Homogeneous Equations with Constant Coefficients	75
2.4	Nonhomogeneous Equations: Methods of Undetermind Coefficients	83
2.4.1	The Method of Variation of Constants (Parameters)	89
2.5	Limiting Behavior of Solutions	91
2.6	Nonlinear Equations Transformable to Linear Equations .	98
2.7	Applications	104
2.7.1	Propagation of Annual Plants	104
2.7.2	Gambler's Ruin	107
2.7.3	National Income	108
2.7.4	The Transmission of Information	110
3	Systems of Linear Difference Equations	117
3.1	Autonomous (Time-Invariant) Systems	117
3.1.1	The Discrete Analogue of the Putzer Algorithm . .	118
3.1.2	The Development of the Algorithm for A^n	119
3.2	The Basic Theory	125
3.3	The Jordan Form: Autonomous (Time-Invariant) Systems Revisited	135
3.3.1	Diagonalizable Matrices	135
3.3.2	The Jordan Form	142
3.3.3	Block-Diagonal Matrices	148
3.4	Linear Periodic Systems	153
3.5	Applications	159
3.5.1	Markov Chains	159
3.5.2	Regular Markov Chains	160
3.5.3	Absorbing Markov Chains	163
3.5.4	A Trade Model	165
3.5.5	The Heat Equation	167
4	Stability Theory	173
4.1	A Norm of a Matrix	174
4.2	Notions of Stability	176

4.3	Stability of Linear Systems	184
4.3.1	Nonautonomous Linear Systems	184
4.3.2	Autonomous Linear Systems	186
4.4	Phase Space Analysis	194
4.5	Liapunov's Direct, or Second, Method	204
4.6	Stability by Linear Approximation	219
4.7	Applications	229
4.7.1	One Species with Two Age Classes	229
4.7.2	Host-Parasitoid Systems	232
4.7.3	A Business Cycle Model	233
4.7.4	The Nicholson-Bailey Model	235
4.7.5	The Flour Beetle Case Study	238
5	Higher-Order Scalar Difference Equations	245
5.1	Linear Scalar Equations	246
5.2	Sufficient Conditions for Stability	251
5.3	Stability via Linearization	256
5.4	Global Stability of Nonlinear Equations	261
5.5	Applications	268
5.5.1	Flour Beetles	268
5.5.2	A Mosquito Model	270
6	The Z-Transform Method and Volterra Difference Equations	273
6.1	Definitions and Examples	274
6.1.1	Properties of the Z -Transform	277
6.2	The Inverse Z -Transform and Solutions of Difference Equations	282
6.2.1	The Power Series Method	282
6.2.2	The Partial Fractions Method	283
6.2.3	The Inversion Integral Method	287
6.3	Volterra Difference Equations of Convolution Type: The Scalar Case	291
6.4	Explicit Criteria for Stability of Volterra Equations	295
6.5	Volterra Systems	299
6.6	A Variation of Constants Formula	305
6.7	The Z -Transform Versus the Laplace Transform	308
7	Oscillation Theory	313
7.1	Three-Term Difference Equations	313
7.2	Self-Adjoint Second-Order Equations	320
7.3	Nonlinear Difference Equations	327
8	Asymptotic Behavior of Difference Equations	335
8.1	Tools of Approximation	335
8.2	Poincaré's Theorem	340

8.2.1	Infinite Products and Perron's Example	344
8.3	Asymptotically Diagonal Systems	351
8.4	High-Order Difference Equations	360
8.5	Second-Order Difference Equations	369
8.5.1	A Generalization of the Poincaré–Perron Theorem	372
8.6	Birkhoff's Theorem	377
8.7	Nonlinear Difference Equations	382
8.8	Extensions of the Poincaré and Perron Theorems	387
8.8.1	An Extension of Perron's Second Theorem	387
8.8.2	Poincaré's Theorem Revisited	389
9	Applications to Continued Fractions and Orthogonal Polynomials	397
9.1	Continued Fractions: Fundamental Recurrence Formula	397
9.2	Convergence of Continued Fractions	400
9.3	Continued Fractions and Infinite Series	408
9.4	Classical Orthogonal Polynomials	413
9.5	The Fundamental Recurrence Formula for Orthogonal Polynomials	417
9.6	Minimal Solutions, Continued Fractions, and Orthogonal Polynomials	421
10	Control Theory	429
10.1	Introduction	429
10.1.1	Discrete Equivalents for Continuous Systems	431
10.2	Controllability	432
10.2.1	Controllability Canonical Forms	439
10.3	Observability	446
10.3.1	Observability Canonical Forms	453
10.4	Stabilization by State Feedback (Design via Pole Placement)	457
10.4.1	Stabilization of Nonlinear Systems by Feedback	463
10.5	Observers	467
10.5.1	Eigenvalue Separation Theorem	468
A	Stability of Nonhyperbolic Fixed Points of Maps on the Real Line	477
A.1	Local Stability of Nonoscillatory Nonhyperbolic Maps	477
A.2	Local Stability of Oscillatory Nonhyperbolic Maps	479
A.2.1	Results with $g(x)$	479
B	The Vandermonde Matrix	481
C	Stability of Nondifferentiable Maps	483

D	Stable Manifold and the Hartman–Grobman–Cushing Theorems	487
D.1	The Stable Manifold Theorem	487
D.2	The Hartman–Grobman–Cushing Theorem	489
E	The Levin–May Theorem	491
F	Classical Orthogonal Polynomials	499
G	Identities and Formulas	501
	Answers and Hints to Selected Problems	503
	Maple Programs	517
	References	523
	Index	531