



<http://www.springer.com/978-0-387-22215-8>

The Geometry of Syzygies

A Second Course in Algebraic Geometry and Commutative
Algebra

Eisenbud, D.

2005, XIV, 246 p., Hardcover

ISBN: 978-0-387-22215-8

Contents

| | |
|---|-----------|
| Preface: Algebra and Geometry | ix |
| What Are Syzygies? | x |
| The Geometric Content of Syzygies | xi |
| What Does Solving Linear Equations Mean? | xii |
| Experiment and Computation | xiii |
| What's In This Book? | xiv |
| Prerequisites | xv |
| How Did This Book Come About? | xv |
| Other Books | xvi |
| Thanks | xvi |
| Notation | xvi |
| 1 Free Resolutions and Hilbert Functions | 1 |
| The Generation of Invariants | 1 |
| Enter Hilbert | 2 |
| 1A The Study of Syzygies | 3 |
| The Hilbert Function Becomes Polynomial | 4 |
| 1B Minimal Free Resolutions | 5 |
| Describing Resolutions: Betti Diagrams | 7 |
| Properties of the Graded Betti Numbers | 8 |
| The Information in the Hilbert Function | 9 |
| 1C Exercises | 10 |
| 2 First Examples of Free Resolutions | 15 |
| 2A Monomial Ideals and Simplicial Complexes | 15 |
| Simplicial Complexes | 15 |
| Labeling by Monomials | 16 |
| Syzygies of Monomial Ideals | 18 |

| | | |
|----------|---|------------|
| 2B | Bounds on Betti Numbers and Proof of Hilbert's Syzygy Theorem . . . | 20 |
| 2C | Geometry from Syzygies: Seven Points in \mathbb{P}^3 | 22 |
| | The Hilbert Polynomial and Function. | 23 |
| | . . . and Other Information in the Resolution | 24 |
| 2D | Exercises | 27 |
| 3 | Points in \mathbb{P}^2 | 31 |
| 3A | The Ideal of a Finite Set of Points | 32 |
| 3B | Examples | 39 |
| 3C | Existence of Sets of Points with Given Invariants | 42 |
| 3D | Exercises | 47 |
| 4 | Castelnuovo–Mumford Regularity | 55 |
| 4A | Definition and First Applications | 55 |
| 4B | Characterizations of Regularity: Cohomology | 58 |
| 4C | The Regularity of a Cohen–Macaulay Module | 65 |
| 4D | The Regularity of a Coherent Sheaf | 67 |
| 4E | Exercises | 68 |
| 5 | The Regularity of Projective Curves | 73 |
| 5A | A General Regularity Conjecture | 73 |
| 5B | Proof of the Gruson–Lazarsfeld–Peskine Theorem | 75 |
| 5C | Exercises | 85 |
| 6 | Linear Series and 1-Generic Matrices | 89 |
| 6A | Rational Normal Curves | 90 |
| | 6A.1 Where'd That Matrix Come From? | 91 |
| 6B | 1-Generic Matrices | 92 |
| 6C | Linear Series | 95 |
| 6D | Elliptic Normal Curves | 103 |
| 6E | Exercises | 113 |
| 7 | Linear Complexes and the Linear Syzygy Theorem | 119 |
| 7A | Linear Syzygies | 120 |
| 7B | The Bernstein–Gelfand–Gelfand Correspondence | 124 |
| 7C | Exterior Minors and Annihilators | 130 |
| 7D | Proof of the Linear Syzygy Theorem | 135 |
| 7E | More about the Exterior Algebra and BGG | 136 |
| 7F | Exercises | 143 |
| 8 | Curves of High Degree | 145 |
| 8A | The Cohen–Macaulay Property | 146 |
| | 8A.1 The Restricted Tautological Bundle | 148 |
| 8B | Strands of the Resolution | 153 |
| | 8B.1 The Cubic Strand | 155 |
| | 8B.2 The Quadratic Strand | 159 |
| 8C | Conjectures and Problems | 169 |
| 8D | Exercises | 171 |

| | |
|---|------------|
| 9 Clifford Index and Canonical Embedding | 177 |
| 9A The Cohen–Macaulay Property and the Clifford Index | 177 |
| 9B Green’s Conjecture | 180 |
| 9C Exercises | 185 |
| Appendix 1 Introduction to Local Cohomology | 187 |
| A1A Definitions and Tools | 187 |
| A1B Local Cohomology and Sheaf Cohomology | 195 |
| A1C Vanishing and Nonvanishing Theorems | 198 |
| A1D Exercises | 199 |
| Appendix 2 A Jog Through Commutative Algebra | 201 |
| A2A Associated Primes and Primary Decomposition | 202 |
| A2B Dimension and Depth | 205 |
| A2C Projective Dimension and Regular Local Rings | 208 |
| A2D Normalization: Resolution of Singularities for Curves | 210 |
| A2E The Cohen–Macaulay Property | 213 |
| A2F The Koszul Complex | 217 |
| A2G Fitting Ideals and Other Determinantal Ideals | 220 |
| A2H The Eagon–Northcott Complex and Scrolls | 222 |
| References | 227 |
| Index | 237 |