

Yu. V. Egorov A. I. Komech
M. A. Shubin

Elements
of the Modern Theory
of Partial Differential
Equations



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I. Linear Partial Differential Equations. Elements of the Modern Theory

Yu.V. Egorov, M.A. Shubin

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