

David E. Edmunds
W. Desmond Evans

Hardy Operators, Function Spaces and Embeddings

With 6 Figures

 Springer

David E. Edmunds

Department of Mathematics
Sussex University
Brighton BN1 9RF, United Kingdom
e-mail: d.e.edmunds@sussex.ac.uk

W. Desmond Evans

School of Mathematics
Cardiff University
Cardiff CF24 4YH, United Kingdom
e-mail: EvansWD@cardiff.ac.uk

Library of Congress Control Number: 2004108695

The cover figure is taken from a paper by L.E. Fraenkel and is Fig. 5.5 on page 232 of the text.

Mathematics Subject Classification (2000):

26D10, 26D15, 34L20, 35J05, 35P20, 45D05, 45P05, 46B50, 46E35, 47B06, 47B10

ISSN 1439-7382

ISBN 978-3-642-06027-4 ISBN 978-3-662-07731-3 (eBook)

DOI 10.1007/978-3-662-07731-3

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag Berlin Heidelberg GmbH. Violations are liable for prosecution under the German Copyright Law.

springeronline.com

© Springer-Verlag Berlin Heidelberg 2004

Originally published by Springer-Verlag Berlin Heidelberg New York in 2004

Softcover reprint of the hardcover 1st edition 2004

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Typeset by the authors using a Springer \LaTeX macro package

Production: LE- \TeX Jelonek, Schmidt & Vöckler GbR, Leipzig

Cover design: Erich Kirchner, Heidelberg

Printed on acid-free paper SPIN: 10973073 41/3142YL - 5 4 3 2 1 0

Contents

1	Preliminaries	1
1.1	Hausdorff and Minkowski dimensions	1
1.2	The area and coarea formulae	3
1.3	Approximation numbers	6
1.4	Inequalities	9
2	Hardy-type Operators	11
2.1	Introduction	11
2.2	Boundedness of T	12
2.3	Compactness of T	17
2.4	Approximation numbers of T	23
2.4.1	The Hardy operator on a finite interval	24
2.4.2	The general case: Preliminaries	31
2.4.3	Estimates for $a_m(T)$, $1 < p \leq q < \infty$	39
2.4.4	Estimates for $a_n(T)$ when $p = 1$ or $q = \infty$	42
2.4.5	Approximation numbers of T when $1 \leq q < p \leq \infty$	43
2.4.6	Asymptotic results for $p = q \in (1, \infty)$	43
2.4.7	The cases $p = 1, \infty$	50
2.5	l_α and $l_{\alpha,w}$ classes.	51
2.6	Hardy-type operators on trees	55
2.6.1	Analysis on trees	55
2.6.2	Boundedness of T	57
2.7	Compactness of T and its approximation numbers	58
2.8	Notes	59
3	Banach function spaces	63
3.1	Introduction	63
3.1.1	Definitions	64
3.2	Rearrangements	69
3.3	Rearrangement-invariant spaces	84
3.4	Examples	90

3.4.1	Lorentz, Lorentz-Zygmund and generalised Lorentz-Zygmund spaces	90
3.4.2	Orlicz spaces	96
3.4.3	Lorentz-Karamata spaces	108
3.4.4	Decompositions	121
3.5	Operators of joint weak type	125
3.5.1	Definitions	125
3.5.2	Operators of strong and weak type	128
3.6	Bessel-Lorentz-Karamata-potential spaces	133
3.6.1	Abstract Sobolev spaces	133
3.6.2	Bessel-Lorentz-Karamata-potential spaces	134
3.6.3	Sub-limiting embeddings	139
3.6.4	Limiting embeddings	140
3.6.5	Super-limiting embeddings	144
3.7	Examples	152
3.8	Other spaces	155
3.9	Notes	158
4	Poincaré and Hardy inequalities	161
4.1	Introduction	161
4.2	Poincaré inequalities in BFSs	164
4.2.1	Poincaré and Friedrichs inequalities	164
4.2.2	Examples	174
4.2.3	Higher-order cases	183
4.3	Concrete spaces	185
4.3.1	Classes of domains	185
4.3.2	Sobolev and Poincaré inequalities	193
4.4	Hardy inequalities	207
4.5	Notes	217
5	Generalised ridged domains	219
5.1	Introduction	219
5.1.1	Ridges and skeletons	220
5.1.2	Simple ridges in \mathbb{R}^2	224
5.2	Generalised ridged domains	228
5.3	Measure of non-compactness	234
5.4	Analysis on GRD	244
5.4.1	The map T and its approximate inverse M	245
5.4.2	Equivalent embeddings	249
5.4.3	Equivalent Poincaré inequalities	251
5.5	Compactness of E	252
5.5.1	Local compactness	252
5.5.2	Measure of non-compactness	254
5.6	Embedding Theorems	261

5.7 The Poincaré inequality and $\alpha(E)$ 266

5.8 Notes 273

6 Approximation numbers of Sobolev embeddings 275

6.1 Introduction 275

6.2 Some quotient space norms 277

6.3 Dirichlet-Neumann bracketing in L_p 282

6.4 Further asymptotic estimates for a GRD Ω 294

6.5 Notes 305

References 307

Author Index 319

Subject Index 323

Notation Index 325