

Wolfgang Ebeling

Lattices and Codes

A Course Partially Based on Lectures by F. Hirzebruch

2nd Revised Edition



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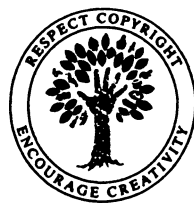
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