Undergraduate Texts in Mathematics

Undergraduate Texts in Mathematics

Anglin: Mathematics: A Concise History and Philosophy.
Readings in Mathematics.
Apostol: Introduction to Analytic Number Theory. Second edition.
Armstrong: Groups and Symmetry.
Armstrong: Basic Topology.
Bak/Newman: Complex Analysis.
Banchoff/Wermer: Linear Algebra Through Geometry. Second edition.
Berberian: A First Course in Real Analysis.
Brémaud: An Introduction to Probabilistic Modeling.
Bressoud: Factorization and Primality Testing.
Bressoud: Second Year Calculus.
Readings in Mathematics.
Brickman: Mathematical Introduction to Linear Programming and Game Theory.
Cederberg: A Course in Modern Geometries.
Childs: A Concrete Introduction to Higher Algebra.
Chung: Elementary Probability Theory with Stochastic Processes. Third edition.
Cox/Little/O'Shea: Ideals, Varieties, and Algorithms.
Curtis: Linear Algebra: An Introductory Approach. Fourth edition.
Devlin: The Joy of Sets: Fundamentals of Contemporary Set Theory. Second edition.
Dixmier: General Topology.
Driver: Why Math?
Ebbinghaus/Flum/Thomas: Mathematical Logic. Second edition.
Edgar: Measure, Topology, and Fractal Geometry.
Fischer: Intermediate Real Analysis.
Flanigan/Kazdan: Calculus Two: Linear and Nonlinear Functions. Second edition.
Fleming: Functions of Several Variables. Second edition.
Foulds: Optimization Techniques: An Introduction.
Foulds: Combinatorial Optimization for Undergraduates.
Franklin: Methods of Mathematical Economics.
Halmos: Finite-Dimensional Vector Spaces. Second edition.
Halmos: Naive Set Theory.
Hämmerlin/Hoffmann: Numerical Mathematics.
Readings in Mathematics.
Iooss/Joseph: Elementary Stability and Bifurcation Theory. Second edition.
James: Topological and Uniform Spaces.
Jänich: Topology.
Kemeny/Snell: Finite Markov Chains.
Klambauer: Aspects of Calculus.
Kinsey: Topology of Surfaces.
Lang: A First Course in Calculus. Fifth edition.
Lang: Calculus of Several Variables. Third edition.
Lang: Introduction to Linear Algebra. Second edition.
Lang: Linear Algebra. Third edition.
Lang: Undergraduate Algebra. Second edition.
Lang: Undergraduate Analysis.

(continued after index)

H.-D. Ebbinghaus
J. Flum
W. Thomas

Mathematical Logic

Second Edition

With 13 Illustrations

Springer Science+Business Media, LLC

H.-D. Ebbinghaus
J. Flum
Mathematisches Institut
Universität Freiburg
Albertstrasse 23b
7800 Freiburg
Germany

W. Thomas
Institut für Informatik und
Praktische Mathematik
Universität Kiel
D 24098 Kiel
Germany

Translated from *Einführung in die mathematische Logik*, published by Wissenschaftliche Buchgesellschaft, Darmstadt, by Ann S. Ferebee, Kohlweg 12, D-6240 Königstein 4, Germany.

(First published in the series: "Die Mathematik. Einführungen in Gegenstand und Ergebnisse ihrer Teilgebiete und Nachbarwissenschaften.")

AMS Subject Classification (1991): 03-01

Library of Congress Cataloging-in-Publication Data
Ebbinghaus, Heinz-Dieter, 1939-
[Einführung in die mathematische Logik.]
Mathematical logic / H.-D. Ebbinghaus, J. Flum, W. Thomas.
p. cm. -- (Undergraduate Texts in Mathematics)
Includes bibliographical references and index.
ISBN 978-1-4757-2357-1 ISBN 978-1-4757-2355-7 (eBook)
DOI 10.1007/978-1-4757-2355-7

1. Logic, Symbolic and mathematical. I. Flum, Jörg.
II. Thomas, Wolfgang, 1947- . III. Title. IV. Series.
QA9.E2213 1994
511.3--dc20 93-50621

Printed on acid-free paper.

Originally published by Springer-Verlag New York, Inc. in 1994
Softcover reprint of the hardcover 2nd edition 1994

Production managed by Jim Harbison; manufacturing supervised by Vincent Scelta.
Camera-ready copy provided by the authors using Springer-Verlag's LaTeX macro svsing.sty.

9 8 7 6 5 4 3 2 1

ISBN 978-1-4757-2357-1

Preface

What is a mathematical proof? How can proofs be justified? Are there limitations to provability? To what extent can machines carry out mathematical proofs?

Only in this century has there been success in obtaining substantial and satisfactory answers. The present book contains a systematic discussion of these results. The investigations are centered around first-order logic. Our first goal is Gödel's completeness theorem, which shows that the consequence relation coincides with formal provability: By means of a calculus consisting of simple formal inference rules, one can obtain all consequences of a given axiom system (and in particular, imitate all mathematical proofs).

A short digression into model theory will help us to analyze the expressive power of the first-order language, and it will turn out that there are certain deficiencies. For example, the first-order language does not allow the formulation of an adequate axiom system for arithmetic or analysis. On the other hand, this difficulty can be overcome—even in the framework of first-order logic—by developing mathematics in set-theoretic terms. We explain the prerequisites from set theory necessary for this purpose and then treat the subtle relation between logic and set theory in a thorough manner.

Gödel's incompleteness theorems are presented in connection with several related results (such as Trahtenbrot's theorem) which all exemplify the limitations of machine-oriented proof methods. The notions of computability theory that are relevant to this discussion are given in detail. The concept of computability is made precise by means of the register machine as a computer model.

We use the methods developed in the proof of Gödel's completeness theorem to discuss Herbrand's Theorem. This theorem is the starting point for a detailed description of the theoretical fundamentals of logic programming. The corresponding resolution method is first introduced on the level of propositional logic.

The deficiencies in expressive power of the first-order language are a motivation to look for stronger logical systems. In this context we introduce, among others, the second-order language and the infinitary languages. For each of them we prove that central facts which hold for the first-order language are no longer valid. Finally, this empirical fact is confirmed by Lindström's theorems, which show that there is no logical system that extends first-order logic and at the same time shares all its advantages.

The book does not require special mathematical knowledge; however, it presupposes an acquaintance with mathematical reasoning as acquired, for example, in the first year of a mathematics or computer science curriculum.

Margit Meßmer prepared the English translation of the extended German edition and the LaTeX–version of the book. We wish to thank her for her efficient and diligent work. For additional LaTeX–editing thanks are due to A. Miller and O. Matz. For helpful suggestions and/or careful proof–reading we also thank U. Bosse, G. Geisler, H. Imhof and J. C. Martinez.

Freiburg and Kiel, July 1993

H.-D. Ebbinghaus
J. Flum
W. Thomas

Contents