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In Cooperation with Volker Peckhaus

# Ernst Zermelo

An Approach to His Life and Work

With 42 Illustrations

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