

P.P.G. Dyke

An Introduction to Laplace Transforms and Fourier Series

With 51 Figures



Springer

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