

William G. Dwyer
Hans-Werner Henn

Homotopy Theoretic Methods in Group Cohomology

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Authors' addresses:

William G. Dwyer
Department of Mathematics
University of Notre Dame
Notre Dame, IN 46556
USA

e-mail: dwyer.1@nd.edu

Hans-Werner Henn
Département de Mathématiques
UFR de Mathématiques et Informatique
7 rue René Descartes
67084 Strasbourg Cedex
France
e-mail: henn@irma.u-strasbg.fr

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Contents

Preface	ix
Classifying Spaces and Homology Decompositions	
<i>William G. Dwyer</i>	
1 Introduction	1
2 Classifying spaces	2
3 Simplicial complexes and simplicial sets	6
4 Simplicial spaces and homotopy colimits	16
5 Nerves of categories and the Grothendieck construction	22
6 Homotopy orbit spaces	26
7 Homology decompositions	28
8 Sharp homology decompositions. Examples	33
9 Reinterpreting the homotopy colimit spectral sequence	36
10 Bredon homology and the transfer	40
11 Acyclicity for G -spaces	43
12 Non-identity p -subgroups	45
13 Elementary abelian p -subgroups	46
14 Appendix	49
References	51

**Cohomology of Groups and Unstable Modules
over the Steenrod Algebra**

Hans-Werner Henn

0	Introduction	55
1	Cohomology of groups	57
1.1	Some fundamental structural results for cohomology groups and cohomology rings of finite groups	57
1.2	Quillen's F -isomorphism theorem	58
2	Quillen's F -isomorphism theorem from the point of view of unstable modules	63
2.1	Unstable modules over the Steenrod algebra	63
2.2	Quillen's F -isomorphism theorem from the point of view of unstable modules	66
3	Lannes' T -functor and Lannes' generalization of Quillen's theorem	68
3.1	Review of Lannes' T -functor	68
3.2	Lannes' generalization of Quillen's theorem	71
3.3	An outline of a proof of Quillen's Theorem	74
4	Approximations up to higher nilpotency	75
4.1	Localizations in abelian categories	75
4.2	Localization away from n -nilpotent modules	77
5	Commutative algebra of unstable H^*BG -modules; approximations of H^*BG up to torsion modules	83
6	The centralizer spectral sequence	90
	References	96