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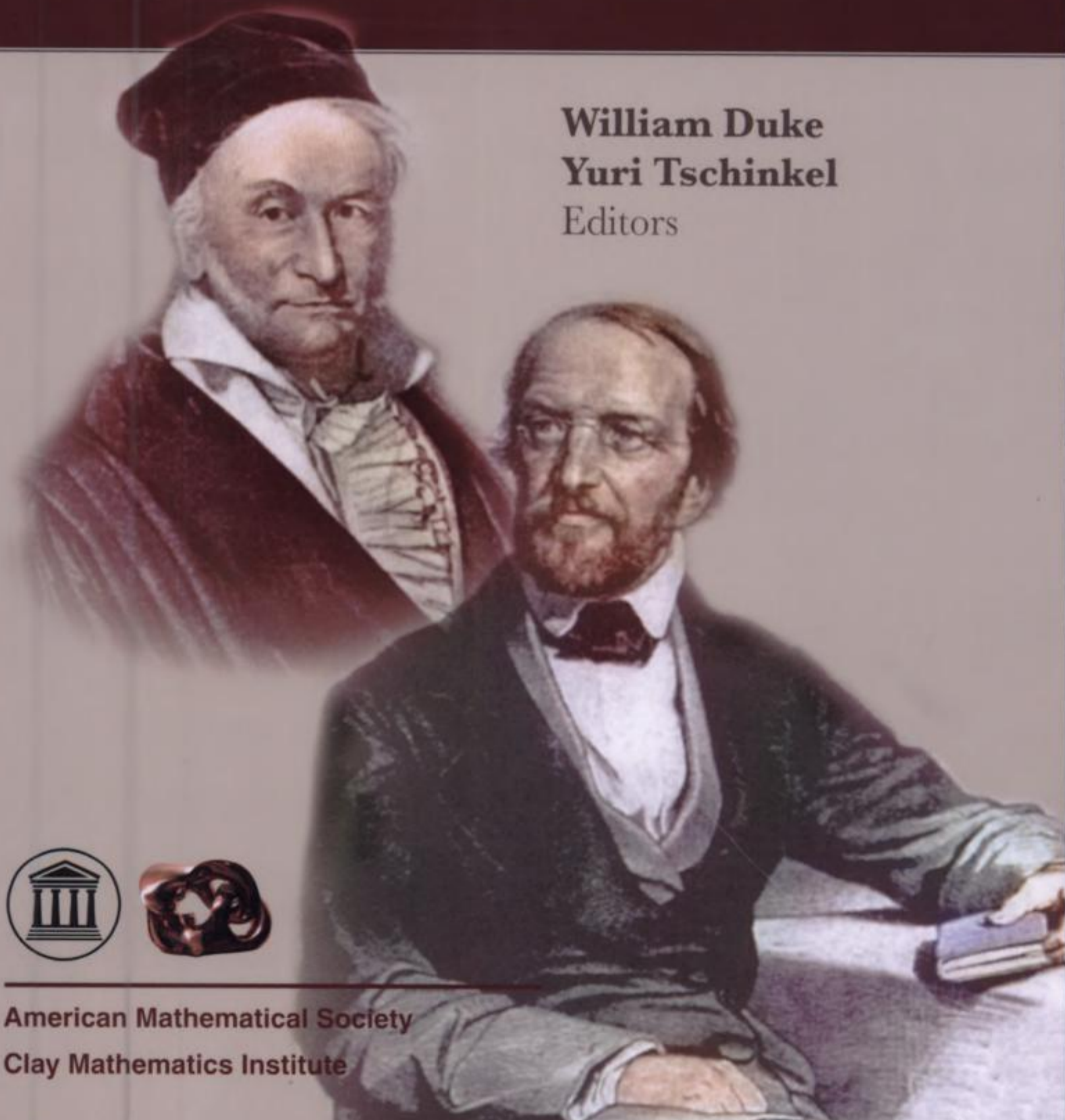
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# **Analytic Number Theory**

## **A Tribute to Gauss and Dirichlet**

**William Duke**  
**Yuri Tschinkel**  
Editors



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The cover photo of Gustav-Peter Lejeune Dirichlet is courtesy of the Muhlenberg College Library.

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