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# Graduate Texts in Mathematics



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J.L. Doob

# Measure Theory

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# Introduction

This book was planned originally not as a work to be published, but as an excuse to buy a computer, incidentally to give me a chance to organize my own ideas on what measure theory every would-be analyst should learn, and to detail my approach to the subject. When it turned out that Springer-Verlag thought that the point of view in the book had general interest and offered to publish it, I was forced to try to write more clearly and search for errors. The search was productive.

Readers will observe the stress on the following points.

**The application of pseudometric spaces**. Pseudometric, rather than metric spaces, are applied to obviate the artificial replacement of functions by equivalence classes, a replacement that makes the use of "almost everywhere" either improper or artificial. The words "function" and "the set on which a function has values at least ε" can be taken literally in this book. Pseudometric space properties are applied in many contexts. For example, outer measures are used to pseudometrize classes of sets and the extension of a finite measure from an algebra to a σ algebra is thereby reduced to finding the closure of a subset of a pseudometric space.

**Probability concepts are introduced in their appropriate place, not consigned to a ghetto**. Mathematical probability is an important part of measure theory, and every student of measure theory should be acquainted with the fundamental concepts and function relations specific to this part. Moreover, probability offers a wide range of measure theoretic examples and applications both in and outside pure mathematics. At an elementary level, probability-inspired examples free students from the delusions that product measures are the only important multidimensional measures and that continuous distributions are the only important distributions. At a more sophisticated level, it is absurd that analysts should be familiar with mutual orthogonality but not with mutual independence of functions, that they should be familiar with theorems on convergence of series of orthogonal functions but not on convergence of martingales.

**Convergence of sequences of measures** is treated both in the general Vitali-Hahn-Saks setting and in the mathematical setting of Borel measures on the metric spaces of classical analysis: the compact metric spaces and the locally compact separable metric spaces. The general discussion is applied in detail to finite Lebesgue-Stieltjes measures on the line, in particular to probability measures.

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