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Well-Posed Optimization Problems

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To our parents, and
To Dora, Mira, Kiril,
Orietta and Guido

PREFACE

This book aims to present, in a unified way, some basic aspects of the mathematical theory of well-posedness in scalar optimization.

The first fundamental concept in this area is inspired by the classical idea of J. Hadamard, which goes back to the beginning of this century. It requires existence and uniqueness of the optimal solution together with continuous dependence on the problem's data.

In the early sixties A. Tykhonov introduced another concept of well-posedness imposing convergence of every minimizing sequence to the unique minimum point. Its relevance to (and motivation from) the approximate (numerical) solution of optimization problems is clear.

In the book we study both the Tykhonov and the Hadamard concepts of well-posedness, the links between them and also some extensions (e.g. relaxing the uniqueness).

Both the pure and the applied sides of our topic are presented. The first four chapters are devoted to abstract optimization problems. Applications to optimal control, calculus of variations and mathematical programming are the subject matter of the remaining five chapters.

Chapter I contains the basic facts about Tykhonov well-posedness and its generalizations. The main metric, topological and differential characterizations are discussed. The Tykhonov regularization method is outlined.

Chapter II is the key chapter (as we see from its introduction) because it is devoted to a basic issue: the relationships between Tykhonov and Hadamard well-posedness. We emphasize the fundamental links between the two concepts in the framework of best approximation problems, convex functions and variational inequalities.

Chapter III approaches the generic nature of well-posedness (or sometimes ill-posedness) within various topological settings. Parametric optimization problems which are well-posed for a dense, or generic, set of parameters are considered. The relationship with differentiability (sensitivity analysis) is pointed out.

Chapter IV establishes the links between Hadamard well-posedness and variational or epi-convergences. In this way several characterizations of Hadamard well-posedness in optimization are obtained. For convex problems the well-posedness is characterized via the Euler-Lagrange equation. An application to nonsmooth problems is presented, and the role of the convergence in the sense of Mosco is exploited, especially for quadratic problems.

Chapter V is the first one devoted to applications of the theory developed in the first four chapters. Characterizations of well-posedness in optimal control problems for ordinary (or partial) differential equations are discussed. We deal with various forms of well-posedness, including Lipschitz properties of the optimal state and control.

Chapter VI discusses the equivalence between the relaxability of optimal control problems and the continuity of the optimal value (with an abstract generalization). The link with the convergence of discrete-time approximations is presented.

Chapter VII focuses on the study of singular perturbation phenomena in optimal control from the point of view of Hadamard well-posedness. Continuity properties of various mappings appearing in singularly perturbed problems (e.g. the reachable set depending on a small parameter in the derivative) are studied.

Chapter VIII is devoted to characterizations of Tykhonov and Hadamard well-posedness for Lagrange problems with constraints in the calculus of variations, after treating integral functionals without derivatives. We also discuss the classical Ritz method, least squares, and the Lavrentiev phenomenon.

Chapter IX considers first the basic (Berge-type) well-posedness results in a topological setting, for abstract mathematical programming problems depending on a parameter. Then we characterize the stability of the feasible set defined by inequalities, via constraint qualification conditions; Lipschitz properties of solutions to generalized equations are also discussed. Hadamard well-posedness in convex mathematical programming is studied. Quantitative estimates for the optimal solutions are obtained using local Hausdorff distances. Results about Lipschitz continuity of solutions in nonlinear and linear programming end the chapter.

We have made an attempt to unify, simplify and relate many scattered results in the literature. Some new results and new proofs are included. We do not intend to deal with the theory in the most general setting; our goal is to present the main problems, ideas and results in as natural a way as possible.

Each chapter begins with an introduction devoted to examples and motivations or to a simple model problem in order to illustrate the specific topic. The formal statements are often introduced by heuristics, particular cases and examples, while the complete proofs are usually collected at the end of each section and given in full detail, even when elementary. Each chapter contains notes and bibliographical remarks.

The prerequisites for reading this book do not extend in general beyond standard real and functional analysis, general topology and basic optimization theory. Some topics occasionally require more special knowledge that is always either referenced or explicitly recalled when needed.

Some sections of this book are based in part on former lecture notes (by T. Zolezzi) under the title "Perturbations and approximations of minimum problems".

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