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Domain Decomposition Methods for the Numerical Solution of Partial Differential Equations

With 40 Figures and 1 Table

 Springer

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