

Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

1190

Optimization and Related Fields

Proceedings of the "G. Stampacchia International
School of Mathematics" held at Erice, Sicily
September 17–30, 1984

Edited by R. Conti, E. De Giorgi and F. Giannessi



Springer-Verlag
Berlin Heidelberg New York Tokyo

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Mathematics Subject Classification (1980): 49-XX, 65KXX, 90CXX

ISBN 3-540-16476-6 Springer-Verlag Berlin Heidelberg New York Tokyo
ISBN 0-387-16476-6 Springer-Verlag New York Heidelberg Berlin Tokyo

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Printed in Germany

Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr.
2146/3140-543210

Preface

This volume contains the Proceedings of an International School of Mathematics devoted to Optimization and Related Fields. Held from 17 to 30 September 1984, the site for the meeting was the "Ettore Majorana" Centre for Scientific Culture in Erice, Sicily. The course was conceived and directed by R.Conti, E.De Giorgi and F.Giannessi as an opportunity for promoting the exchange and for stimulating the interaction among the various branches of optimization, like calculus of variations, theory of optimal controls, mathematical programming, convex analysis, and so on. Such branches have now a fast development around the world, so that a titer interaction seems to be interesting and useful.

The school was attended by 67 people from 12 countries. In the tradition of the Ettore Majorana Centre for Scientific Culture, the participants were classified either as "invited speakers" or as "students", though in fact the latter were not students in the ordinary sense; most of them were Ph.D. holders with established publication records. Several of them contributed to the course with the following seminars:

- On point-to-set mappings in discrete programming and some applications by B.Bank;*
- Stability and existence in nonlinear mixed-integer programming by B. Bank;*
- Some relaxation results for integral functionals by G.Buttazzo;*
- On a stability problem in Γ -convergence by R. De Arcangelis;*
- Relaxation in the optimal control of abstract evolution inclusions by C.Engblom;*
- Necessary conditions for constrained optimization problems: some remarks on Dubovitskii-Miljutin theory by P.Favati and S.Steffè;*
- Existence and approximation of optimal solutions of infinite horizon*

programs by S.D.Flam;

- Existence results and stability in Pareto problems by R.Lucchetti;
- Closure and upper semi-continuity results in mathematical programming, Nash economic equilibria by R.Lucchetti and F.Patrone;
- Stable monotone variational inequalities by L.McLinden;
- Regularity conditions for constrained extremum problems by L.Martein;
- Generalized Bellman equation for the optimal time problem in Hilbert Space by F.Mignanego;
- Efficiency and optimality of allocations in large squares economies by N.S.Papageorgiou;
- Lagrangian duality in nonlinear programming by M.Pappalardo;
- On the image of a constrained extremum problem by F.Tardella;
- Duality gap: an upper bound and a characterization of the problems with zero gap by F.Tardella;
- Non-coercive variational problems and applications to elasticity by F.Tomarelli;
- Jacobi sufficiency criterion for optimal control by V.Zeidan.

We express our sincere thanks to all those who took part in the school. Special mention should be made of the Ettore Majorana Centre in Erice which offered its facilities and stimulating environment for the meeting. We are all indebted to the Italian National Research Council (C.N.R.), and Ministry of Public Education for their financial support. Finally we want to express our special thanks to Springer-Verlag for their unfailing cooperation.

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