SERIES IN ALGEBRA VOLUME 2

SYLOW THEORY, FORMATIONS AND FITTING CLASSES IN LOCALLY FINITE GROUPS

Martyn R. Dixon

Department of Mathematics
University of Alabama
USA

World Scientific
Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 9128

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 73 Lynton Mead, Totteridge, London N20 8DH

Library of Congress Cataloging-in-Publication Data

Dixon, Martyn R. (Martyn Russell), 1955–
Sylow theory, formations, and fitting classes in locally finite groups / Martyn R. Dixon.
p. cm. -- (Series in algebra ; v. 2)
Includes bibliographical references.
ISBN 9810217951
1. Finite groups. 2. Sylow subgroups. I. Title. II. Series: Series in algebra ; vol. 2.
QA177.D59 1994
512'.2--dc20 94-30313
CIP

Copyright © 1994 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 27 Congress Street, Salem, MA 01970, USA.

Printed in Singapore by Uto-Print

CONTENTS

Preface v

Index of Notation xi

1. Basic concepts 1
- 1.1 Group theoretical classes and closure operations 1
- 1.2. Series of subgroups 6
- 1.3. Radicals and residuals 13
- 1.4. Local systems 18
- 1.5. The minimum condition and Černikov groups 27
- 1.6. Generalizations of the minimum condition 35
- 1.7. Some miscellaneous results 46

2. Sylow theory in locally finite groups 53
- 2.1. Introduction 53
- 2.2. Elementary results and examples 54
- 2.3. Conjugacy and the size of $|\mathrm{Syl}_\pi G|$ 65
- 2.4. The Asar-Hartley Theorem for a general set of primes 76
- 2.5. Groups with min-p 82
- 2.6 Good Sylow subgroups 93

3. Groups satisfying min-p for all primes p 99
- 3.1. Sylow theory in groups with min-p for all primes p 99
- 3.2. Locally soluble groups of finite rank 104
- 3.3. Some properties of $PSL(2, F)$ 110
- 3.4. The 2-radicable part of a group with min-p for all p 114
- 3.5. The structure of groups with min-p for all primes p 125

4. Groups with conjugate Sylow subgroups 141
- 4.1. Upper π-separable groups 141
- 4.2. Groups with the minimum condition on centralizers 144
- 4.3. The structure of Sylow p-integrated groups 158
- 4.4. Completely Sylow integrated groups 160
- 4.5. Metabelian groups with min-n 165

5. Sylow bases in locally finite groups 169
5.1. General properties of Sylow bases 169
5.2. Sylow bases in $\mathfrak{U}$-groups 175
5.3. Sylow bases in groups with min-p for all p 181
5.4. Further results on groups with min-p for all p 188
5.5. Co-Hopfian groups 196

6. Formation theory in locally finite groups 205
6.1. A brief historical account 205
6.2. Locally defined formations 207
6.3. The $\mathfrak{F}$-normalizers of locally finite groups 216
6.4. $\mathfrak{F}$-projectors in locally finite groups 230
6.5. Concluding remarks concerning formation theory 245

7. Fitting classes in locally finite groups 251
7.1 A brief historical account 251
7.2. Elementary properties of Fitting classes 252
7.3. Injectors in locally finite groups 257
7.4. Injectors in groups with min-p for all primes p 265
7.5. Conjugacy conditions for injectors in $\mathfrak{L}$-groups 272
7.6. Closing remarks 280

Bibliography 283

Index 299