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continued after Index

Jacques Dixmier

General Topology

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Jacques Dixmier
Université de Paris VI
Mathématiques UER 48
9 Quai Saint Bernard
75 Paris
France

Translated from the French by
Sterling K. Berberian
Department of Mathematics
The University of Texas
Austin, TX 78712
U.S.A.

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Contents

Introduction

This book is a course in general topology, intended for students in the first year of the second cycle (in other words, students in their third university year). The course was taught during the first semester of the 1979–80 academic year (three hours a week of lecture, four hours a week of guided work).

Topology is the study of the notions of limit and continuity and thus is, in principle, very ancient. However, we shall limit ourselves to the origins of the theory since the nineteenth century. One of the sources of topology is the effort to clarify the theory of real-valued functions of a real variable: uniform continuity, uniform convergence, equicontinuity, Bolzano–Weierstrass theorem (this work is historically inseparable from the attempts to *define* with precision what the real numbers are). Cauchy was one of the pioneers in this direction, but the errors that slip into his work prove how hard it was to isolate the right concepts. Cantor came along a bit later; his researches into trigonometric series led him to study in detail sets of points of **R** (whence the concepts of open set and closed set in **R**, which in his work are intermingled with much subtler concepts).

The foregoing alone does not justify the very general framework in which this course is set. The fact is that the concepts mentioned above have shown themselves to be useful for objects other than the real numbers. First of all, since the nineteenth century, for points of $\mathbf{R}^n$. Next, especially in the twentieth century, in a good many other sets: the set of lines in a plane, the set of linear transformations in a real vector space, the group of rotations, the Lorentz group, etc. Then in 'infinite-dimensional' sets: the set of all continuous functions, the set of all vector fields, etc.

Topology divides into 'general topology' (of which this course exposes the rudiments) and 'algebraic topology', which is based on general topology but makes use of a lot of algebra. We cite some theorems whose most natural proofs appeal to algebraic topology:

(1) let B be a closed ball in $\mathbf{R}^n$, f a continuous mapping of B into B; then f has a fixed point;
(2) for every $x \in \mathbf{S}_2$ (the 2-dimensional sphere) let $\vec{V}(x)$ be a vector tangent to $\mathbf{S}_2$ at x; suppose that $\vec{V}(x)$ depends continuously on x; then there exists an $x_0 \in \mathbf{S}_2$ such that $\vec{V}(x_0) = 0$;
(3) let U and V be homeomorphic subsets of $\mathbf{R}^n$; if U is open in $\mathbf{R}^n$, then V is open in $\mathbf{R}^n$.

These theorems cannot be obtained by the methods of this course, but, having seen their statements, some readers will perhaps want to learn something about algebraic topology.

The sign ► in the margin pertains to theorems that are especially deep or especially useful. The choice of these statements entails a large measure of arbitrariness: there obviously exist many little remarks, very easy and constantly used, that are not graced by the sign ►.

The sign * signals a passage that is at the limits of 'the program' (by which I mean what has been more or less traditional to teach at this level for some years).

Quite a few of the statements have already been encountered in the First cycle. For clarity and coherence of the text, it seemed preferable to take them up again in detail.

The English edition differs from the French by various minor improvements and by the addition of a section on normal spaces (Chapter 7, Section 6).