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Discrete Mathematical Models in Population Biology

Ecological, Epidemic,
and Evolutionary Dynamics

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