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Graph Theory

Second Edition

With 122 Illustrations

Graph Theory is a comprehensive textbook on the topic. It is intended both for students of mathematics and for computer scientists and engineers. As a textbook it includes complete proofs of all theorems and also contains numerous exercises, many of which have solutions. As a research monograph it contains several results which have not previously appeared in a book, such as the Tutte–Brooks theorem on colorability, the perfect graph conjecture, and the strong perfect graph conjecture. The book also includes a discussion of algorithms, complexity, and NP-completeness.

The second edition has been substantially revised and updated. It includes new chapters on topics such as Ramsey theory, large graphs, and random graphs. It also includes new sections on Szemerédi's regularity lemma and its applications, as well as on the construction of expander graphs. The book is suitable for advanced undergraduate or graduate courses in graph theory, discrete mathematics, or computer science. It can also be used as a reference for researchers in the field.



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Contents

Preface	vii
1. The Basics	1
1.1. Graphs	2
1.2. The degree of a vertex	4
1.3. Paths and cycles	6
1.4. Connectivity	9
1.5. Trees and forests	12
1.6. Bipartite graphs	14
1.7. Contraction and minors	16
1.8. Euler tours	18
1.9. Some linear algebra	20
1.10. Other notions of graphs	25
Exercises	26
Notes	28
2. Matching	29
2.1. Matching in bipartite graphs	29
2.2. Matching in general graphs	34
2.3. Path covers	39
Exercises	40
Notes	42

3. Connectivity	43
3.1. 2-Connected graphs and subgraphs	43
3.2. The structure of 3-connected graphs	45
3.3. Menger's theorem	50
3.4. Mader's theorem	56
3.5. Edge-disjoint spanning trees	58
3.6. Paths between given pairs of vertices	61
Exercises	63
Notes	65
4. Planar Graphs	67
4.1. Topological prerequisites	68
4.2. Plane graphs	70
4.3. Drawings	76
4.4. Planar graphs: Kuratowski's theorem	80
4.5. Algebraic planarity criteria	85
4.6. Plane duality	87
Exercises	89
Notes	92
5. Colouring	95
5.1. Colouring maps and planar graphs	96
5.2. Colouring vertices	98
5.3. Colouring edges	103
5.4. List colouring	105
5.5. Perfect graphs	110
Exercises	117
Notes	120
6. Flows	123
6.1. Circulations	124
6.2. Flows in networks	125
6.3. Group-valued flows	128
6.4. k -Flows for small k	133
6.5. Flow-colouring duality	136
6.6. Tutte's flow conjecture	140
Exercises	144
Notes	145

7. Substructures in Dense Graphs	147
7.1. Subgraphs	148
7.2. Szemerédi's regularity lemma	153
7.3. Applying the regularity lemma	160
Exercises	165
Notes	166
8. Substructures in Sparse Graphs	169
8.1. Topological minors	170
8.2. Minors	179
8.3. Hadwiger's conjecture	181
Exercises	184
Notes	186
9. Ramsey Theory for Graphs	189
9.1. Ramsey's original theorems	190
9.2. Ramsey numbers	193
9.3. Induced Ramsey theorems	197
9.4. Ramsey properties and connectivity	207
Exercises	208
Notes	210
10. Hamilton Cycles	213
10.1. Simple sufficient conditions	213
10.2. Hamilton cycles and degree sequences	216
10.3. Hamilton cycles in the square of a graph	218
Exercises	226
Notes	227
11. Random Graphs	229
11.1. The notion of a random graph	230
11.2. The probabilistic method	235
11.3. Properties of almost all graphs	238
11.4. Threshold functions and second moments	242
Exercises	247
Notes	249

12. Minors, Trees, and WQO	251
12.1. Well-quasi-ordering	251
12.2. The graph minor theorem for trees	253
12.3. Tree-decompositions	255
12.4. Tree-width and forbidden minors	263
12.5. The graph minor theorem	274
Exercises	277
Notes	280
Hints for all the exercises	283
Index	299
Symbol index	311