The Costa surface

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Minimal Surfaces I

Boundary Value Problems

With 172 Figures and 8 Colour Plates

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Preface

This book has grown out of a series of lectures given by the second author at Bonn University. Its topic, belonging both to differential geometry and to the calculus of variations, may at first glance seem rather special. We believe, however, that it is both attractive and advantageous to consider mathematical ideas in the light of special problems, even though mathematicians nowadays tend to prefer the opposite approach, namely to emphasize general theories while relegating specific problems to play the modest role of examples. Both ways to present mathematics are equally valuable and necessary, but the theory of minimal surfaces is a good case for the first approach, to study in some detail examples which are as fascinating as they are important.

Our intention in writing this book is best characterized by a quote from Courant's treatise *Dirichlet's principle* which in several respects has been a model for our work: "*Enlightenment, however, must come from an understanding of motives; live mathematical development springs from specific natural problems which can easily be understood, but whose solutions are difficult and demand new methods of more general significance.*"

The aim of our monograph is to give an introduction to the field of two-dimensional minimal surfaces in Euclidean space with particular emphasis on boundary value problems. To keep the scope of this text within reasonable limits, we decided to omit various interesting topics altogether or to treat them only in an introductory way. In several cases we just provide surveys of the pertinent recent literature. Of the topics we regretfully had to omit we specifically mention the theory of nonparametric minimal surfaces, the investigations on stable minimal surfaces (field construction and eigenvalue criteria for the second variation of the area functional, global properties of stable surfaces) and on unstable solutions of boundary value problems as well as the recent discoveries on complete minimal surfaces. Moreover, results for surfaces of prescribed mean curvature and for minimal surfaces in Riemannian manifolds will not, with only few exceptions, be described, and geometric measure theory is not touched upon at all except for a brief survey of its applications to minimal surfaces. Nor do we provide a comprehensive presentation of the Osserman-Gulliver-Alt theory concerning the nonexistence of true and false branch points for area-minimizing solutions of boundary value problems, although we describe some of its main ideas in two instances.

Thus the reader should not expect an encyclopedic treatment of the theory of minimal surfaces but merely an introduction to the field, followed by a more thorough presentation of certain aspects which relate to boundary value problems. For further study we refer to our extensive bibliography as well as to comments and references in the Scholia attached to each chapter.

Our omissions might in part be excusable since several of the topics neglected are dealt with in the treatises of Courant [15], Federer [1], Nitsche [28], Giusti [4], and in the notes by Osserman [10], Lawson [6], Massari-Miranda [1], Struwe [11], Simon [8] and Jost [17]. The overlap of our notes and Nitsche's monograph is not very large, and in several respects the two books are complementary to each other. Together they provide a fairly comprehensive (but still not complete) picture of the theory of two-dimensional minimal surfaces.

Despite our attempts to limit the present notes in size and to restrain our desire for completeness the manuscript became too extensive for a single handy volume. Therefore we followed the advice of our publisher and split the collected material into two separate volumes (denoted as Minimal Surfaces I and II). The first one is more elementary than the second and may serve as an introduction to the theory of minimal surfaces and to Plateau's problem. To achieve a relatively self-contained presentation of our subject we have included an introductory chapter on the differential geometry of two-dimensional surfaces where we also fix the notation and collect some of the fundamental formulas to be used in later computations; it can be skipped at first reading. We have not included any material on Sobolev spaces since nowadays most students are acquainted with this topic for which our standard source of references is the treatise of Gilbarg-Trudinger [1].

The second volume is more specialized as it is mainly devoted to the study of the boundary behaviour of minimal surfaces satisfying various kinds of boundary conditions. In addition we treat the so-called thread problem, and we also provide an introduction to the general Plateau problem.

Each volume can be read independently of the other, although we use results from Volume 1 in Volume 2 and vice versa. Thus the specialist in the field may like to consult only Volume 2 whereas the nonexpert may just want to read Volume 1. However, since Part II of the first volume offers several comprehensive surveys of more recent work, this part of the book might be of interest also to the initiated.

We would like to mention that a substantial part of our notes originated in joint work of the second author with M. Grüter, E. Heinz, J.C.C. Nitsche, and F. Sauvigny. Other parts are drawn from work by H.W. Alt, R. Courant, J. Douglas, G. Dziuk, R. Gulliver, E. Heinz, W. Jäger, H. Karcher, H. Lewy, C.B. Morrey, J.C.C. Nitsche, R. Osserman, T. Radó, F. Tomi, A. Tromba, and from the authors' own results. A few items are taken from the survey of Radó and from the treatises of Courant and Nitsche.

We are grateful to many colleagues who supported us in writing this book, in particular to Maria Athanassenas, Alfred Baldes, Leung Fu Cheung, Gerhard Dziuk, David Hoffman, Jürgen Jost, Hermann Karcher, Peter Li, and Michael

Struwe. We thank Robert Osserman for providing us with Example [5] in Section 3.7. We are very much indebted to Anthony Tromba who wrote for us the main part of Chapter 11, based on his joint work [4] with Friedrich Tomi, and who also supplied material for the Scholia of Chapter 4. Martin Haneke and Andreas Wirse wrote parts of the programs that we used to draw our illustrations. We would also like to thank David Hoffman, Hermann Karcher, Konrad Polthier, and Meinhard Wohlgemuth for permitting us to use some of their drawings of complete and of periodic minimal surfaces, and Imme Haubitz for permitting us to reproduce some of her drawings of Thomsen surfaces. We are grateful to Klaus Bach, Frei Otto and Eric Pitts for providing us with photographs of various soap film experiments. The support of our work by the Computer Graphics Laboratory of the Institute of Applied Mathematics at Bonn University and of the Sonderforschungsbereich 256 was invaluable. We are especially grateful to Eva Küster who polished both style and grammar of these notes, to Carol and John Weston for advice concerning the usage of the English language, and to Anke Thiedemann who professionally and with untiring patience typed many versions of our manuscript. We should also like to thank many students and colleagues at Bonn University who pointed out errors and misprints at a first stage of the manuscript. In particular we thank Hellai Abdullah, Julia Brettschneider, John Duggan, Christoph Hamburger, Katrin Rhode and Gudrun Turowski. We acknowledge the help of David Hoffman, Hermann Karcher, Friedrich Tomi and Anthony Tromba in reading parts of the galley proofs. Without the generous support of SFB 72 and of SFB 256 at Bonn University this book could not have been written. Last but not least we should like to thank the patient publisher and his collaborators, in particular Joachim Heinze and Karl-Friedrich Koch, for their encouragement and help.

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