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Groups, Trees and Projective Modules

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To the memory
of my mother

PREFACE

For 1978/9 the Ring Theory Study Group at Bedford College rather naively set out to learn what had been done in the preceding decade on groups of cohomological dimension one. This is a particularly attractive subject, that has witnessed substantial success, essentially beginning in 1968 with results of Serre, Stallings and Swan, later receiving impetus from the introduction of the concept of the fundamental group of a connected graph of groups by Bass and Serre, and recently culminating in Dunwoody's contribution which completed the characterization. Without going into definitions, one can state the result simply enough: For any nonzero ring R (associative, with 1) and group G, the augmentation ideal of the group ring R[G] is right R[G]-projective if and only if G is the fundamental group of a graph of finite groups having order invertible in R.

These notes, a (completely) revised version of those prepared for the Study Group, collect together material from several sources to present a self-contained proof of this fact, assuming at the outset only the most elementary knowledge - free groups, projective modules, etc. By making the rôle of derivations even more central to the subject than ever before, we were able to simplify some of the existing proofs, and in the process obtain a more general "relativized" version of Dunwoody's result, cf IV.2.10. An amusing outcome of this approach is that we here have a proof of one of the major results in the theory of cohomology of groups that nowhere mentions cohomology - which should make this account palatable to hard-line ring theorists. (Group theorists will notice we have not touched upon the fascinating subject of ends of groups, usually one of the cornerstones of this topic, cf Cohen [72]; happily, an up-to-date outline of the subject of ends is available in the recently published lecture notes of Scott-Wall [79].)

There are four chapters. Chapter I covers, in the first six sections, the basics of the Bass-Serre theory of groups acting on trees (using derivations to prove the key theorem, I.5.3), and then in I.§8, I.§9 gives an abstract treatment of Dunwoody's results on groups acting on partially ordered sets with involution. Chapter II gives the standard classical applications of the Bass-Serre theory, including a proof of Higgins' generalization of the Grushko-Neumann theorem (based on a proof by I.M.Chiswell). Chapter III presents the Dunwoody-Stallings decomposition of a group arising from a derivation to a projective module, and gives Dunwoody's accessibility criteria. Finally, in Chapter IV, the groups of cohomological dimension one are introduced and characterized; the final section describes the basic consequences for finite extensions of free groups.

A reader interested mainly in the projectivity results of IV.§2 can pursue the following course:Chapter I:§§1-6,§8,§9; Chapter II:3.1,3.3,3.5; Chapter III:1.1,1.2,§2,§3,4.1-4.8,4.11, Chapter IV:§1,§2.

Since the subject is quite young, and the notation to some extent still tentative, we have felt at liberty to introduce new terminology and notation wherever it suited our needs, or satisfied our category-theoretic prejudices. At these points, we have made an effort to indicate the notations used by other authors.

PREFACE

Through ignorance, we have been unable to give much in the way of historical remarks, and those we have given may be inaccurate, since, as both Cohen and Scott have remarked, it is difficult to attribute, with any precision, results which existed implicitly in the literature before being made explicit.

The computer microfilm drawings, pp 13, 25, were produced by the CDC 7600 at the University of London Computer Centre, using their copyrighted software package DIMFILM. I thank Chris Cookson and Phil Taylor for their helpful technical advice in using this package.

I thank all the participants of the Study Group for their kind indulgence in this project, and especially Yuri Bahturin and Bill Stephenson for relieving me (and the audience) by giving many of the seminars.

I gratefully acknowledge much useful background material (and encouragement) from the experts at Queen Mary College, I.M. Chiswell and D.E. Cohen for Chapters I-II and III-IV respectively.

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NOTATION AND CONVENTIONS

The following notation will be used:

ϕ	for the empty set;
$\mathbb{Z}$	for the ring of integers;
$\mathbb{Q}$	for the field of rational numbers;
$\mathbb{C}$	for the field of complex numbers;
$A - B$	for the set of elements in A not in B;
$\|A\|$	for the cardinal of A;
B^A	for the set of all functions from A to B, the elements thought of as A-tuples with entries chosen from B;
$A \times B$, $\prod_{\alpha \in A} B_\alpha$	for the Cartesian product;
$A \vee B$, $\bigvee_{\alpha \in A} B_\alpha$	for the disjoint union of sets;
$A \oplus B$, $\bigoplus_{\alpha \in A} B_\alpha$	for the direct sum of modules.

Functions are usually, but not always, written on the right of their arguments.

All theorems, propositions, lemmas, corollaries, remarks and conventions are numbered consecutively in each section, thus 4.3 CONVENTION follows 4.2 DEFINITION in section I.4 (and outside Chapter I they are referred to as I.4.3 and I.4.2). The end of each subsection is indicated by $\square$.

References to the bibliography are by author's name and the last two digits of the year of publication, thus Serre [77], with primes to distinguish publications by the same author in the same year.

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