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# Numerical Analysis in Modern Scientific Computing

An Introduction

Second Edition

With 65 Illustrations



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