Table of Contents

Outline of Contents 1

1 Introduction 7
1.1 Newton-Raphson Method for Scalar Equations 7
1.2 Newton's Method for General Nonlinear Problems 11
1.2.1 Classical convergence theorems revisited 11
1.2.2 Affine invariance and Lipschitz conditions 13
1.2.3 The algorithmic paradigm 20
1.3 A Roadmap of Newton-type Methods 21
1.4 Adaptive Inner Solvers for Inexact Newton Methods 26
1.4.1 Residual norm minimization: `GMRES` 28
1.4.2 Energy norm minimization: `PCG` 30
1.4.3 Error norm minimization: `CGNE` 32
1.4.4 Error norm reduction: `GBIT` 35
1.4.5 Linear multigrid methods 38
Exercises 40

Part I ALGEBRAIC EQUATIONS

2 Systems of Equations: Local Newton Methods 45
2.1 Error Oriented Algorithms 45
2.1.1 Ordinary Newton method 45
2.1.2 Simplified Newton method 52
2.1.3 Newton-like methods 56
2.1.4 Broyden's 'good' rank-1 updates 58
2.1.5 Inexact Newton-ERR methods 67
2.2 Residual Based Algorithms 76
2.2.1 Ordinary Newton method 76

2.2.2 Simplified Newton method . . . 79
2.2.3 Broyden's 'bad' rank-1 updates . . . 81
2.2.4 Inexact Newton-RES method . . . 90
2.3 Convex Optimization . . . 94
2.3.1 Ordinary Newton method . . . 94
2.3.2 Simplified Newton method . . . 97
2.3.3 Inexact Newton-PCG method . . . 98
Exercises . . . 104

3 Systems of Equations: Global Newton Methods . . . 109
3.1 Globalization Concepts . . . 110
3.1.1 Componentwise convex mappings . . . 111
3.1.2 Steepest descent methods . . . 114
3.1.3 Trust region concepts . . . 117
3.1.4 Newton path . . . 121
3.2 Residual Based Descent . . . 125
3.2.1 Affine contravariant convergence analysis . . . 126
3.2.2 Adaptive trust region strategies . . . 128
3.2.3 Inexact Newton-RES method . . . 131
3.3 Error Oriented Descent . . . 134
3.3.1 General level functions . . . 135
3.3.2 Natural level function . . . 138
3.3.3 Adaptive trust region strategies . . . 145
3.3.4 Inexact Newton-ERR methods . . . 152
3.4 Convex Functional Descent . . . 161
3.4.1 Affine conjugate convergence analysis . . . 162
3.4.2 Adaptive trust region strategies . . . 165
3.4.3 Inexact Newton-PCG method . . . 168
Exercises . . . 170

4 Least Squares Problems: Gauss-Newton Methods . . . 173
4.1 Linear Least Squares Problems . . . 175
4.1.1 Unconstrained problems . . . 175
4.1.2 Equality constrained problems . . . 178
4.2 Residual Based Algorithms . . . 182
4.2.1 Local Gauss-Newton methods . . . 183
4.2.2 Global Gauss-Newton methods . . . 188

4.2.3 Adaptive trust region strategy 190
4.3 Error Oriented Algorithms 193
4.3.1 Local convergence results 194
4.3.2 Local Gauss-Newton algorithms 197
4.3.3 Global convergence results 205
4.3.4 Adaptive trust region strategies 212
4.3.5 Adaptive rank strategies 215
4.4 Underdetermined Systems of Equations 221
4.4.1 Local quasi-Gauss-Newton method 221
4.4.2 Global Gauss-Newton method 225
Exercises 228

5 Parameter Dependent Systems: Continuation Methods . . . 233
5.1 Newton Continuation Methods 237
5.1.1 Classification of continuation methods 237
5.1.2 Affine covariant feasible stepsizes 242
5.1.3 Adaptive pathfollowing algorithms 245
5.2 Gauss-Newton Continuation Method 250
5.2.1 Discrete tangent continuation beyond turning points . . 250
5.2.2 Affine covariant feasible stepsizes 253
5.2.3 Adaptive stepsize control 257
5.3 Computation of Simple Bifurcations 263
5.3.1 Augmented systems for critical points 263
5.3.2 Newton-like algorithm for simple bifurcations 271
5.3.3 Branching-off algorithm 277
Exercises 280

Part II DIFFERENTIAL EQUATIONS

6 Stiff ODE Initial Value Problems 285
6.1 Affine Similar Linear Contractivity 285
6.2 Nonstiff versus Stiff Initial Value Problems 288
6.2.1 Picard iteration versus Newton iteration 288
6.2.2 Newton-type uniqueness theorems 291
6.3 Uniqueness Theorems for Implicit One-step Methods 295
6.4 Pseudo-transient Continuation for Steady State Problems 299

6.4.1 Exact pseudo-transient continuation 300
6.4.2 Inexact pseudo-transient continuation 307
Exercises 312

7 ODE Boundary Value Problems 315
7.1 Multiple Shooting for Timelike BVPs 316
7.1.1 Cyclic linear systems 318
7.1.2 Realization of Newton methods 324
7.1.3 Realization of continuation methods 327
7.2 Parameter Identification in ODEs 333
7.3 Periodic Orbit Computation 337
7.3.1 Single orbit computation 337
7.3.2 Orbit continuation methods 340
7.3.3 Fourier collocation method 344
7.4 Polynomial Collocation for Spacelike BVPs 349
7.4.1 Discrete versus continuous solutions 350
7.4.2 Quasilinearization as inexact Newton method 356
Exercises 366

8 PDE Boundary Value Problems 369
8.1 Asymptotic Mesh Independence 369
8.2 Global Discrete Newton Methods 378
8.2.1 General PDEs 378
8.2.2 Elliptic PDEs 385
8.3 Inexact Newton Multilevel FEM for Elliptic PDEs 389
8.3.1 Local Newton-Galerkin methods 391
8.3.2 Global Newton-Galerkin methods 397
Exercises 403

References 405

Software 416

Index 419