Peter Dembowski

# Finite Geometries

Reprint of the 1968 Edition

Springer

Peter Dembowski †

Originally published as Vol. 44 of the
*Ergebnisse der Mathematik und ihrer Grenzgebiete*

Mathematics Subject Classification (1991): 51E, 52

CIP data applied for

Die Deutsche Bibliothek – CIP-Einheitsaufnahme

**Dembowski, Peter:**
Finite geometries / Peter Dembowski. - Reprint of the 1968 ed. - Berlin; Heidelberg; New York; Barcelona; Budapest; Hong Kong; London; Milan; Paris; Santa Clara; Singapore; Tokyo: Springer, 1997
(Classics in mathematics)

ISBN-13: 978-3-540-61786-0 e-ISBN-13: 978-3-642-62012-6
DOI: 10.1007/978-3-642-62012-6

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1997

The use of general descriptive names, registered names, trademarks etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

SPIN 10554514 41/3143-5 4 3 2 1 0 – Printed on acid-free paper

# Contents

**Note on exposition** . . . . . XI

**1. Basic concepts** . . . . . 1

1.1 Finite incidence structures . . . . . 1
1.2 Incidence preserving maps . . . . . 8
1.3 Incidence matrices . . . . . 17
1.4 Geometry of finite vector spaces . . . . . 23

**2. Designs** . . . . . 56

2.1 Combinatorial properties . . . . . 56
2.2 Embeddings and extensions . . . . . 69
2.3 Automorphisms of designs . . . . . 78
2.4 Construction of designs . . . . . 92

**3. Projective and affine planes** . . . . . 115

3.1 General results . . . . . 115
3.2 Combinatorics of finite planes . . . . . 137
3.3 Correlations and polarities . . . . . 151
3.4 Projectivities . . . . . 157

**4. Collineations of finite planes** . . . . . 169

4.1 Fixed elements and orders . . . . . 169
4.2 Collineation groups . . . . . 178
4.3 Central collineations . . . . . 187
4.4 Groups with few orbits . . . . . 207

**5. Construction of finite planes** . . . . . 219

5.1 Algebraic representations . . . . . 219
5.2 Planes of type IV . . . . . 228
5.3 Planes of type V . . . . . 236
5.4 Planes of types I and II . . . . . 246

**6. Inversive planes** . . . . . 252

6.1 General definitions and results . . . . . 252
6.2 Combinatorics of finite inversive planes . . . . . 262
6.3 Automorphisms . . . . . 268
6.4 The known finite models . . . . . 273

**7. Appendices** . . . . . 281

7.1 Association schemes and partial designs . . . . . 281
7.2 Hjelmslev planes . . . . . 291
7.3 Generalized polygons . . . . . 300
7.4 Finite semi-planes . . . . . 305

**Bibliography** . . . . . 318

**Dictionary** . . . . . 367

**Special notations** . . . . . 369

**Index** . . . . . 371