

Pierre Del Moral

Feynman-Kac Formulae

Genealogical and Interacting
Particle Systems with Applications

With 15 Illustrations



Springer

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