



Camillo De Lellis

**Rectifiable Sets, Densities
and Tangent Measures**



European Mathematical Society

Prof. Camillo De Lellis
Institut für Mathematik
Universität Zürich
Winterthurerstrasse 190
CH-8057 Zürich
Switzerland

2000 Mathematics Subject Classification (primary; secondary) 28A75; 26B15, 49Q15, 49Q20.

ISBN 978-3-03719-044-9

The Swiss National Library lists this publication in The Swiss Book, the Swiss national bibliography, and the detailed bibliographic data are available on the Internet at <http://www.helveticat.ch>.

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in other ways, and storage in data banks. For any kind of use permission of the copyright owner must be obtained.

© 2008 European Mathematical Society

Contact address:

European Mathematical Society Publishing House
Seminar for Applied Mathematics
ETH-Zentrum FLI C4
CH-8092 Zürich
Switzerland

Phone: +41 (0)44 632 34 36
Email: info@ems-ph.org
Homepage: www.ems-ph.org

Printed on acid-free paper produced from chlorine-free pulp. TCF ∞

Printed in Germany

9 8 7 6 5 4 3 2 1

Contents

1	Introduction	1
2	Notation and preliminaries	4
2.1	General notation and measures	4
2.2	Weak* convergence of measures	5
2.3	Covering theorems and differentiation of measures	9
2.4	Hausdorff measures	10
2.5	Lipschitz functions	11
2.6	The Stone–Weierstrass Theorem	12
3	Marstrand’s Theorem and tangent measures	13
3.1	Tangent measures and Proposition 3.4	16
3.2	Lemma 3.7 and some easy remarks	21
3.3	Proof of Lemma 3.8	22
3.4	Proof of Corollary 3.9	25
4	Rectifiability	27
4.1	The Area Formula I: Preliminary lemmas	29
4.2	The Area Formula II	32
4.3	The Geometric Lemma and the Rectifiability Criterion	35
4.4	Proof of Theorem 4.8	37
5	The Marstrand–Mattila Rectifiability Criterion	40
5.1	Preliminaries: Purely unrectifiable sets and projections	42
5.2	The proof of the Marstrand–Mattila rectifiability criterion	47
5.3	Proof of Theorem 5.1	53
6	An overview of Preiss’ proof	56
6.1	The cone $\{x_4^2 = x_1^2 + x_2^2 + x_3^2\}$,	59
6.2	Part A of Preiss’ strategy	63
6.3	Part B of Preiss’ strategy: Three main steps	65
6.4	From the three main steps to the proof of Theorem 6.10	66
7	Moments and uniqueness of the tangent measure at infinity	70
7.1	From Proposition 7.7 to the uniqueness of the tangent measure at infinity	74
7.2	Elementary bounds on $b_{k,s}$ and the expansion (7.5)	76
7.3	Proof of Proposition 7.7	79

8	Flat versus curved at infinity	85
8.1	The tangent measure at infinity is a cone	88
8.2	Conical uniform measures	88
8.3	Proof of Proposition 8.5	91
9	Flatness at infinity implies flatness	95
9.1	Proofs of (ii) and (iv)	99
9.2	An integral formula for $\text{tr}(b_2^{(2)} \llcorner V)$	100
9.3	An intermediate inequality	103
9.4	Proof of (9.7) and conclusion	106
10	Open problems	110
Appendix A.	Proof of Theorem 3.11	117
Appendix B.	Gaussian integrals	122
<u>Bibliography</u>	<u>125</u>
<u>Index</u>	<u>127</u>