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Heat Conduction Within Linear Thermoelasticity

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Contents

Introduction

J-B. J. FOURIER's immensely influential treatise *Théorie Analytique de la Chaleur* [21], and the subsequent developments and refinements of FOURIER's ideas and methods at the hands of many authors, provide a highly successful theory of heat conduction.

According to that theory, the growth or decay of the temperature θ in a conducting body is governed by the *heat equation*, that is, by the parabolic partial differential equation

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}=\frac{\partial\theta}{\partial t}.$$

Such has been the influence of FOURIER's theory, which must forever remain the *classical* theory in that it sets the standard against which all other theories are to be measured, that the mathematical investigation of heat conduction has come to be regarded as being almost identical† with the study of the heat equation, and the reader will not need to be reminded that intensive analytical study has

† But not entirely; witness, for example, those theories which would replace the heat equation by an equation which implies a finite speed of propagation for the temperature. The reader is referred to the article [9] of COLEMAN, FABRIZIO, and OWEN for the derivation of such an equation from modern Continuum Thermodynamics and for references to earlier work in this direction.

amply demonstrated that the heat equation enjoys many properties of great interest and elegance.†

The arguments upon which the derivation of the heat equation is based presume the conducting body to be *rigid*, and, thus, they ignore any possible interaction between thermal effects and mechanical effects. It is the purpose of this tract to suggest that insight into the nature of thermomechanical interaction can be obtained by studying what is a very restricted subject indeed, namely heat conduction according to the one-dimensional version of the equations of linear thermoelasticity for a homogeneous and isotropic body. These equations constitute the simplest generalization of the heat equation which incorporates the effect of thermomechanical coupling and the effect of inertia. At all points we shall attempt to point out both the contrasts and the similarities between the heat equation and the thermoelastic equations.

The tract does not pretend to be a systematic or complete account of linear thermoelasticity, and, indeed, it is difficult to see how such an account could be written at the present time, for the investigation of thermomechanical interaction is not sufficiently far advanced. In saying this I intend no disparagement of such standard works as those of BOLEY and WEINER [4], CARLSON [6], CHADWICK [8], or NOWACKI [29], but I would point out that what is known of the implications of the coupled dynamic theory is slight by comparison with what is known of the implications of the uncoupled or equilibrium theories.

† Detailed accounts of the heat equation are to be found in the treatises of CANNON [5], CARSLAW and JAEGER [7], and WIDDER [32].