

Table of Contents

| | |
|---|--------------------|
| Preface | ix |
| Notation and Conventions | xi |
| PART I: Background Information and the Statements of the Main Results | |
| Chapter 1. Reviews of Various Topics | 3 |
| 1.1 Review from geometric measure theory | 3 |
| 1.2 Review of some topics concerning singular integral operators and rectifiability | 7 |
| 1.3 Review of some aspects of Littlewood-Paley theory in connection with rectifiability | 16 |
| 1.4 Various characterizations of uniform rectifiability | 21 |
| 1.5 The weak geometric lemma and its relatives | 26 |
| Chapter 2. A Summary of the Main Results | 31 |
| 2.1 The results of Part II | 31 |
| 2.2 Bilateral approximation from a functorial point of view | 37 |
| 2.3 The results of Part III | 42 |
| 2.4 A rapid description of Part IV | 50 |
| Chapter 3. Dyadic Cubes and Corona Decompositions | 53 |
| 3.1 Cubes | 53 |
| 3.2 Corona decompositions | 55 |
| 3.3 Generalized corona decompositions | 63 |
| PART II: New Geometrical Conditions Related to Uniform Rectifiability | |
| Chapter 1. One-Dimensional Sets | 69 |
| 1.1 The weak connectedness condition | 69 |
| 1.2 The weaker local symmetry condition ($d = 1$) | 77 |
| 1.3 Weak constant density for one-dimensional sets | 86 |
| 1.4 The weak “two points on spheres” condition | 93 |

| | |
|--|------------|
| Chapter 2. The Bilateral Weak Geometric Lemma and its Variants | 97 |
| 2.1 Introduction; the corona method | 97 |
| 2.2 Big projections in codimension 1 | 104 |
| 2.3 Big projections in the higher codimension case | 110 |
| 2.4 The local convexity condition LCV | 120 |
| 2.5 The weaker local convexity condition WLCV | 126 |
| 2.6 Weak starlikeness | 129 |
| 2.7 Some questions about variants of the LCV and the LS | 131 |
| Chapter 3. The WHIP and Related Conditions | 135 |
| 3.1 The WHIP, the WTP, and uniform rectifiability | 135 |
| 3.2 The WHIP and weaker versions of the BWGL | 138 |
| 3.3 The weak exterior convexity condition and the GWEC | 141 |
| 3.4 The weak-no-mugs, weak-no-boxes, and weak-no-reels conditions | 147 |
| 3.5 The proof of Theorem 3.9 (part 1) | 154 |
| 3.6 Part 2 of the proof: The stopping-time argument | 165 |
| Chapter 4. Other Conditions in the Codimension 1 Case | 183 |
| 4.1 Introduction | 183 |
| 4.2 Labellings | 187 |
| 4.3 The derivation of Theorem 4.9 from Theorem 4.31 | 196 |
| PART III: Applications | |
| Chapter 1. Uniform Rectifiability and Singular Integral Operators | 207 |
| 1.1 Preliminaries | 207 |
| 1.2 Step one | 208 |
| 1.3 Step two | 212 |
| 1.4 An abstraction of §3 | 214 |
| Chapter 2. Uniform Rectifiability and Square Function Estimates for the Cauchy Kernel | 217 |
| 2.1 Some general comments about square function estimates | 217 |
| 2.2 Uniform rectifiability implies the USFE when $d = 1$ | 219 |
| 2.3 From square function estimates to uniform rectifiability: Preliminary reductions and the plan of the proof | 226 |
| 2.4 The proof of Lemma 2.36 | 229 |
| 2.5 A topological lemma | 232 |
| 2.6 The main step in the proof of Proposition 2.38 | 234 |
| 2.7 The end of the proof of Proposition 2.38 | 244 |
| Chapter 3. Square Function Estimates and Uniform Rectifiability in Higher Dimensions | 249 |
| 3.1 A brief review of Clifford analysis | 249 |
| 3.2 Clifford analysis and square function estimates | 251 |

| | |
|---|---------------------|
| 3.3 From square functions to uniform rectifiability: Preliminary reductions | 252 |
| 3.4 Cauchy flatness implies rectifiability | 253 |
| 3.5 The analogue of Proposition 2.59 | 256 |
| 3.6 Cauchy flatness implies weak flatness | 261 |
| 3.7 Weak flatness implies exterior convexity | 265 |
| 3.8 Some remarks about the higher-codimension case | 267 |
| Chapter 4. Approximating Lipschitz Functions by Affine Functions | 269 |
| 4.1 The direct estimates | 269 |
| 4.2 The converse when $d = 1$ | 279 |
| 4.3 A more abstract version of the WALA | 293 |
| Chapter 5. The Weak Constant Density Condition | 297 |
| 5.1 Compactness will only get you so far | 297 |
| 5.2 The codimension 1 case, part 1 | 301 |
| 5.3 A general lemma about Carleson packing conditions | 305 |
| 5.4 The codimension 1 case, part 2 | 306 |
| 5.5 The weak dyadic density condition | 307 |
| PART IV: Direct Arguments for Some Stability Results | |
| Chapter 1. Stability of Various Versions of the Geometric Lemma | 313 |
| 1.1 The statements | 313 |
| 1.2 A John-Nirenberg-Strömberg lemma for Carleson packing conditions | 315 |
| 1.3 Two lemmas on approximations of regular sets by d -planes | 318 |
| 1.4 The proof of the theorems | 324 |
| Chapter 2. Stability Properties of the Corona Decomposition | 327 |
| 2.1 Corona decompositions revisited | 327 |
| 2.2 Corona constructions and Lipschitz functions | 328 |
| 2.3 The statement of the main result | 336 |
| 2.4 Preliminaries | 336 |
| 2.5 The proof of Lemma 2.38 | 340 |
| References | 345 |
| Table of Selected Notation | 349 |
| Table of Acronyms | 351 |
| Table of Theorems | 353 |
| Index | 355 |