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Mathematical Analysis and Numerical Methods for Science and Technology

Volume 3
Spectral Theory and Applications

With the Collaboration of
Michel Artola and Michel Cessenat

Translated from the French by John C. Amson

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Introduction to Volume 3

This third volume (which contains Chaps. VIII and IX) continues the study of linear stationary boundary value problems and related questions begun in volumes 1 and 2.
The study of the *spectral theory* of elliptic linear operators is fundamental not only to the study of stationary problems in this volume and volume 4, but also to the evolution problems studied in volumes 5 and 6.
In applications, part of the spectrum is often continuous and its treatment is delicate. Such difficulties are familiar, for example, in quantum physics. Here the tools have been introduced which allow the correct treatment of the continuous spectrum.
In a large number of applications (for example in the theory of neutron diffusion) the spectral value "furthest to the right" in the complex plane is real, a situation which corresponds to some concrete physical properties of the system. The study of this furthest right eigenvalue is the object of the Krein-Rutman Theorem presented in the appendix to Chap. VIII.
Examples of the *applications* of the theories in Chap. VIII to electromagnetism and quantum physics are given in Chap. IX.
The operators arising in models are *differential* when they correspond to *local* phenomena (see Chap. V, §1.) Non-local phenomena (for example, action of a force at a distance in space, in electromagnetism; or memory in time in viscoelastic phenomena; or again, abrupt change in a gas particle's velocity in a collision, and the consequential finite variation of velocity in a neutron transport velocity space) cannot be modelled using only these differential operators. In particular, *integral operators* play a large rôle in such models. These models then become *integral equations* (or integro-differential equations, as for example in the case of transport equations; see Chap. I, § 5). The study of their corresponding equations and related stationary problems is continued in the next volume (Volume 4).

The authors of various contributions in each chapter are
Chapter VIII: M. Artola (principal), M. Cessenat.
Chapter IX: M. Cessenat.

Equally we thank P. G. Ciarlet, G. Fournet, R. Glowinski, B. Mercier, P. Raviart, R. Sentis, L. Tartar, H. Viviand for reading certain sections of the text and for their advice.
To M. Cessenat we address our very particular thanks for the permanent and eminent contributions which he has continued to make in this volume 3 as in the previous two volumes 1 and 2, as well as for his detailed and constructive clarifications.

We recall the important and indispensable rôle played by J. M. Moreau of which we listed the various aspects in the preface to volume 1 and for which we will not be able to thank him enough.
The reader wishing to proceed rapidly to the essentials of the mathematical and numerical methods may use this volume 3 by deferring for a later occasion the reading of §4 of Chap. VIII and its appendix, and the whole of Chap. IX (and also the appendix "Singular Integrals" in Volume 4). These parts are distinguished by an asterisk at the appropriate part of the text, and also in table of contents.
We have placed at the end of this volume 3 the *table of notations* used throughout all six volumes.

R. Dautray, J.-L. Lions

Practical Guide for the Reader

1. Designation of subdivisions of the text:
number of a chapter: in Roman numerals
number of major division of a chapter: the sign § followed by a numeral
number of section: a numeral following the above
number of a sub-section: a numeral following the above.

Example: II, §3.5.2, denotes chapter II, §3, section 5, subsection 2.

2. *Within each division* (§), the equations, definitions, theorems, propositions, corollaries, lemmas, remarks and examples are *each* numbered consecutively beginning with the number 1.

3. The *table of notations* used is placed at the end of each volume.

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