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Elliptic Boundary Value Problems on Corner Domains

Smoothness and Asymptotics of Solutions

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FOREWORD

Many physical phenomena are described by elliptic boundary value problems : let us quote vibrating membranes, elasticity, electrostatics, hydrodynamics for instance. Natural domains are often non-smooth ones or they may be "small perturbations" of such non-regular domains. That is why many people are interested in domains with singularities on their boundaries.

In this book, we deal with a great variety of domains : we consider conical singularities of course, but also edges, polyhedral corners, combined with various types of cracks, holes or slits.

In order to give precise mathematical results, we need to choose a functional framework. So we decided, therefore to choose ordinary hilbertian Sobolev spaces with real exponents (also called Sobolev-Slobodeckii spaces). Other choices are possible, but we prefered this one for several reasons that we explain in the introduction .

We develop a general theory : first, we characterize different fundamental properties of induced operators, in particular regularity, Fredholm and semi-Fredholm properties, and then we give asymptotics of solutions in the neighborhood of singular points of the boundary.

Our results can be applied to specific problems : in such cases, it is often possible to get the characteristic conditions we give more precise. As an example, we do this for the Dirichlet problem associated to the Laplace equation. In another paper, we apply them to the Stokes system.

Moreover, the type of statements we get can be adapted to other problems than those we consider here : for instance to non-homogeneous boundary data, to lifting of traces, and also to the study of such problems in other classes of hilbertian weighted Sobolev spaces.

So our results can be used in direct or indirect ways. More introductory details may be found in the preface and in the first section.

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