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A History of Inverse Probability: From Thomas Bayes to Karl Pearson

Andrew I. Dale

A History of Inverse Probability
From Thomas Bayes to Karl Pearson



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To F. J. H.

PROLOCUTION

It will be no strange thing at all for some to dislike the matter of this work, and others to be displeased with the manner and method of it. Easily can I foresee that my account will be too long and tedious for some, while others, perhaps, may be apt to complain of its being too short and concise.

Edmund Calamy

Preface

It is thought as necessary to write a Preface before a Book, as it is judged civil, when you invite a Friend to Dinner, to proffer him a Glass of Hock beforehand for a Whet.

John Arbuthnot, from the preface to his translation of Huygens's "De Ratiociniis in Ludo Aleae".

Prompted by an awareness of the importance of Bayesian ideas in modern statistical theory and practice, I decided some years ago to undertake a study of the development and growth of such ideas. At the time it seemed appropriate to begin such an investigation with an examination of Bayes's *Essay towards solving a problem in the doctrine of chances* and Laplace's *Théorie analytique des probabilités*, and then to pass swiftly on to a brief consideration of other nineteenth century works before turning to what would be the main topic of the treatise, *videlicet* the rise of Bayesian statistics from the 1950's to the present day.

It soon became apparent, however, that the amount of Bayesian work published was such that a thorough investigation of the topic up to the 1980's would require several volumes — and also run the risk of incurring the wrath of extant authors whose writings would no doubt be misrepresented, or at least be so described. It seemed wise, therefore, to restrict the period and the subject under study in some way, and I decided to concentrate my attention on inverse probability from Thomas Bayes to Karl Pearson.

Pearson was born in 1857 and died in 1936, and in a sense a watershed in statistics was reached during his lifetime. The somewhat cavalier approach to inverse probability that one finds in many writings in the century following the publication of Bayes's *Essay* was succeeded in the fullness of time (even if destined only by Tyche) by the logical and personal approach to

probability grounded on the works of Jeffreys, Johnson, Keynes, Ramsey and Wrinch in the first third of this century (and Jeffreys in fact gained his inspiration from Pearson's *Grammar of Science*). At roughly the same time Fisher was making himself a statistical force — indeed, one can perhaps view the rigorous development of Bayes's work into a statistical tool to be reckoned with as a reaction to Fisher's evolution of sampling theory. The thirties also saw the birth of the Neyman-Pearson (and later Wald) decision-theoretic school, and subsequent work of this school was later incorporated into the Bayesian set-up, to the distinct advantage of both.

One must also note the rise of the biometric school, in which Pearson of course played a considerable rôle, and which owed its growth to the appearance of Francis Galton's *Natural Inheritance* of 1889 and his work on correlation. This work also awoke Walter Frank Raphael Weldon's interest in correlation, and he in turn did much to turn Pearson's thoughts to evolution. W.S. Gosset's work c.1908 foreshadowed an attenuation in inverse probability, a tendency which was to be reversed only in the mid-twentieth century.

It would not be too great a violation of the truth to say that, after roughly the beginning of this century, inverse probability took a back seat to the biometric, Fisherian and logical schools, from which it would only rise around 1950 with the work of Good and Savage and the recognition of the relevance of de Finetti's earlier studies. Pearson, whose writings cover both inverse probability and what would today be grouped under "classical" methods, seems then to be a suitable person with whom to end this study.

Todhunter's classic *History of the Mathematical Theory of Probability* was published in 1865. For reasons as to which it would be futile to speculate here, nothing in similar vein, and of such depth, appeared for almost a century (I except books nominally on other topics but containing passages or chapters on the history of statistics or probability, anthologies of papers on this topic, and works on the history of social or political statistics and assurances) until David's little gem of 1962. Several works in similar vein followed, the sequence culminating in Stigler's *History of Statistics* of 1986 and Hald's *History of Probability and Statistics*, the latter appearing in 1990 as the writing of this book nears completion (for trying to write a preface before the actual text is complete is surely as awkward as trying to "squeeze a right-hand foot into a left-hand shoe").

Before I am carelessly castigated or maliciously maligned let me say what will *not* be found here. Firstly, there will be little biographical detail, apart from that in the first chapter on Thomas Bayes. Secondly, little will be found in the way of attempt at putting the various matters discussed in the "correct" historical and sociological context. To interpret early results from

a modern perspective is at best misguided, and I lack the historian's ability, or artifice, to place myself in the period in which these results were first presented. Those interested in these aspects will find abundant satisfaction in the *Dictionary of National Biography*, the *Dictionary of Scientific Biography*, and the books by Hald and Stigler cited above. Daston's *Classical Probability in the Enlightenment* of 1988 may also be useful: like the work by Hald it appeared too late to be consulted in the writing of this text.

Our aim is more modest — and the captious critic will no doubt opine with Winston Churchill that there is much to be modest about! It is to present a record of work on inverse probability (that is, crudely speaking, the arguing from observed events to the probability of causes) over some 150 years from its generally recognized inception to the rise of its sample-theoretic and logical competitors. Since this is a record, it has been thought advisable to preserve the original notations and the languages used — at least almost everywhere. For while translations may well help the thoughtful reader, the serious scholar will need the original text to avoid being misled by the translator's inability to render precise any nuances taxing his linguistic capabilities.

We have not considered only the most important works published during the period under consideration: minor writings, sometimes by seldom cited authors, have also been examined, in order that the effect of the greater works on the wider community of scholars be also noted. It is to be hoped, though, that this consideration has not led to a book of which it can be said, as M.G. Kendall [1963] said of Todhunter's magnum opus, that "it is just about as dull as any book on probability could be."

It is not claimed that this is *the* history of inverse probability: rather, it is one man's view of the topic, a view, it is hoped, in which any peculiarities observed will be ascribed to innocent illusion rather than deliberate delusion, and in which the seeds of future research may be nurtured.

Is there not something essentially diabolical in keeping the impatient reader, even for one moment, from the joys that await him?

D. N. Brereton, introduction to Charles Dickens's "Christmas Books", British Books edition.

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stone's 1941 paper; J.C.B. Mohr (Paul Siebeck), from the second edition of J. von Kries's *Die Principien der Wahrscheinlichkeitsrechnung* (1927); Oxford University Press, from (i) R.A. Fisher's *Statistical Methods and Scientific Inference* (1956) (re-issued by Oxford University Press in 1990), (ii) *The Dictionary of National Biography*, (iii) A.G. Matthews's *Calamy Revised. Being a revision of Edmund Calamy's Account of the ministers ejected and silenced, 1660–1662* (1934) and (iv) F.Y. Edgeworth's papers published in *Mind* in 1884 and 1920; Peter Smith Publishers Inc., from K. Pearson's *The Grammar of Science* (1969 reprint); Princeton University Press, from T.M. Porter's *The Rise of Statistical Thinking* (1986); Springer-Verlag, from my papers of 1982 and 1986, published in the *Archive for History of Exact Sciences*; the Royal Society, from R.A. Fisher's paper published in the *Philosophical Transactions* in 1922; the Royal Statistical Society, from the papers published in the *Journal* of that body by F.Y. Edgeworth (1921), J.D. Holland (1962) and S.M. Stigler (1982); Taylor & Francis, Ltd., from the papers published in *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* by F.Y. Edgeworth (1883, 1884) and K. Pearson (1907); John Wiley & Sons, Ltd., from B. de Finetti's *Probability, Induction and Statistics* (1972). Excerpta from Thomas Bayes's election certificate and G. Boole's "Sketch of a theory and method of probabilities founded upon the calculus of logic" are reproduced by kind permission of the President and Council of the Royal Society of London. Extracts are reprinted from "Note on a Scholium of Bayes", by F.H. Murray, *Bulletin of the American Mathematical Society*, vol. 36, number 2 (February 1930), pp. 129–132, and from "The Theory of Probability: Some Comments on Laplace's *Théorie Analytique*", by E.C. Molina, *Bulletin of the American Mathematical Society*, vol. 36, number 6 (June 1930), pp. 369–392, by permission of the American Mathematical Society. The material quoted from (i) W.L. Harper & C.A. Hooker's *Foundations of Probability Theory, Statistical Inference, and Statistical Theories of Science*, vol. 2 (1976), (ii) J. Hintikka, D. Gruender & E. Agazzi's *Pisa Conference Proceedings*, vol. 2 (1980) and (iii) B. Skyrms & W.L. Harper's *Causation, Chance, and Credence* (1988), is reprinted by permission of Kluwer Academic Publishers.

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The original will of Thomas Bayes is in the custody of the Public Record

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A. I. DALE
Durban, Natal
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