

Ulrich Daepf

Pamela Gorkin

# Reading, Writing, and Proving

*A Closer Look at Mathematics*



Springer

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