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Bernard Dacorogna

Direct Methods in the Calculus of Variations

With 10 Figures

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Preface

In recent years there has been a considerable renewal of interest in the classical problems of the calculus of variations, both from the point of view of mathematics and of applications. Some of the most powerful tools for proving existence of minima for such problems are known as direct methods. They are often the only available ones, particularly for vectorial problems. It is the aim of this book to present them. These methods were introduced by Tonelli, following earlier work of Hilbert and Lebesgue.

Although there are excellent books on calculus of variations and on direct methods, there are recent important developments which cannot be found in these books; in particular, those dealing with vector valued functions and relaxation of non convex problems. These two last ones are important in applications to nonlinear elasticity, optimal design... . In these fields the variational methods are particularly effective. Part of the mathematical developments and of the renewal of interest in these methods finds its motivations in nonlinear elasticity. Moreover, one of the recent important contributions to nonlinear analysis has been the study of the behaviour of nonlinear functionals under various types of convergence, particularly the weak convergence. Two well studied theories have now been developed, namely Γ-convergence and compensated compactness. They both include as a particular case the direct methods of the calculus of variations, but they are also, both, inspired and have as main examples these direct methods.

This monograph is addressed to readers having some elementary notions of functional analysis and Sobolev spaces; however, most of the facts which we use, concerning these notions, are listed and sometimes proved in Chapter 2. Chapter 3 is concerned with minimization problems involving only scalar functions, while Chapter 4 deals with vector valued functions. Finally, in Chapter 5 we study the relaxation of non-convex problems in the scalar as well as the vectorial case. In an appendix we give some applications to nonlinear elasticity and optimal design of the theory developed earlier.

This book is part of a more extended project that originally arose jointly with L. Boccardo. We finally decided to split it into two parts. This monograph corresponds to the first of these and essentially deals with the calculus of variations. The second part will be concerned with nonlinear elliptic partial differential equations and will appear later.

A large part of the present book has been influenced by long discussions with L. Boccardo, particularly as concerns Chapter 3 and the plan of the monograph. Without his collaboration this book would never have been written.

I would like to thank I. Ekeland who, from the beginning, showed enthusiasm for this project and encouraged me strongly to go ahead with it. It was while giving a graduate course in Paris-Dauphine that the project of writing a book originated.

I want to thank J.C. Evard who helped me in writing and in dealing with complicated notations in the Appendix of Chapter 4.

My thanks also go to B. Kawohl who pointed out several mistakes in the manuscript; to P. Ciarlet who communicated to me the manuscript of his recent book which helped me in the writing of the Appendix and of the Bibliography; to J.M. Ball, P. Marcellini, J. Moser, E. Zehnder; to my colleagues in Lausanne C.A. Stuart, B. Zwahlen and to many others with whom I had interesting and helpful discussions.

Particular thanks go to Mrs G. Rime who not only skilfully and rapidly typed the manuscript, but also was patient with all changes and constraints I imposed on her.

Springer-Verlag and the editors of the Applied Mathematical Sciences series have been encouraging and efficient during the process of reviewing and editing this book.

Finally I would like to thank the institutions that supported me during the writing of this book, the Ecole Polytechnique Fédérale de Lausanne, the Université de Paris-Dauphine and the Eidgenössische Technische Hochschule in Zürich.

Lausanne, October 1988 B. DACOROGNA

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