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# Non-Archimedean L-Functions and Arithmetical Siegel Modular Forms

Second, Augmented Edition



Springer

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