Richard Courant

Dirichlet's Principle, Conformal Mapping, and Minimal Surfaces

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DIRICHLET'S PRINCIPLE, CONFORMAL MAPPING, AND MINIMAL SURFACES

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with an Appendix by
M. SCHIFFER

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To

Otto Neugebauer

Preface

It has always been a temptation for mathematicians to present the crystallized product of their thoughts as a deductive general theory and to relegate the individual mathematical phenomenon into the role of an example. The reader who submits to the dogmatic form will be easily indoctrinated. Enlightenment, however, must come from an understanding of motives; live mathematical development springs from specific natural problems which can be easily understood, but whose solutions are difficult and demand new methods of more general significance.

The present book deals with subjects of this category. It is written in a style which, as the author hopes, expresses adequately the balance and tension between the individuality of mathematical objects and the generality of mathematical methods.

The author has been interested in Dirichlet's Principle and its various applications since his days as a student under David Hilbert. Plans for writing a book on these topics were revived when Jesse Douglas' work suggested to him a close connection between Dirichlet's Principle and basic problems concerning minimal surfaces. But war work and other duties intervened; even now, after much delay, the book appears in a much less polished and complete form than the author would have liked.

It was felt desirable to include a report on some recent progress in the theory of conformal mapping: fortunately Professor M. Schiffer, who had a most active part in those developments, agreed to write a summary of the material; the result is the comprehensive appendix which will certainly be considered as a highly valuable contribution to the volume.

In a field which has attracted so many mathematicians it is difficult to achieve a fair accounting of the literature and to appraise merits of others. I have tried to acknowledge all the sources of information and inspiration of which I am conscious, and I hope that not too many omissions have occurred.

A first draft of the book was completed eight years ago, supported by a grant from the Philosophical Society and with the help of

Dr. Wolfgang Wasow. Assistance for the present publication was partly provided under contract with the Office of Naval Research. On the scientific side the book owes much to Professor Max Shiffman, who has been concerned with the theory of minimal surfaces ever since a good fortune brought him as a student to my seminar on the subject. Carl Ludwig Siegel read the manuscript carefully and gave much valuable advice. Avron Douglis, Martin Kruskal, Peter Lax, Imanuel Marx, Joseph Massera, and others have unselfishly devoted time to scrutinizing the manuscript, reading proof, and preparing the bibliography. The drawings were made mainly by George Evans, Jr., Beulah Marx, and Irving Ritter. Edythe Rodermund and Harriet Schoverling gave outstanding secretarial help. The strenuous responsibility for the editorial work and for the supervision of all the steps from preparing the manuscript to the final printing was in the competent hands of Natascha Artin. Without the collective help of all these friends the book could hardly have appeared at this time.

Naturally, a word of thanks must be added for the understanding and patient publisher whose interest has been most encouraging.

The book is dedicated to Otto Neugebauer as a token of friendship and admiration.

R. Courant

New Rochelle, New York
April 1950

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