

B

Progress in Mathematics

Volume 76

Series Editors

J. Oesterlé

A. Weinstein

François R. Cossec
Igor V. Dolgachev

Enriques Surfaces I

1989

Birkhäuser
Boston · Basel · Berlin

François R. Cossec
MATRA
78182 Saint-Quentin-en-Yven
France

Igor V. Dolgachev
Department of Mathematics
University of Michigan
Ann Arbor, MI 48109-1003
U.S.A.

Library of Congress Cataloging-in-Publication Data
Cossec, François R.

Enriques surfaces/François R. Cossec, Igor V. Dolgachev.
p. cm. — (Progress in mathematics ; v. 76—)
Bibliography: v. 1, p.
Includes index.

1. Enriques surfaces. I. Dolgachev, I. (Igor V.) II. Title.
III. Series: Progress in mathematics (Boston, Mass.) ; vol. 76, etc.
QA573.C67 1989
516.3'52—dc19 88-8180

CIP-Titelaufnahme der Deutschen Bibliothek

Cossec, François R.:
Enriques surfaces/François R. Cossec ; Igor V. Dolgachev.—
Boston;Basel:Birkhäuser.
NE: Dolgachev, Igor V.;
1 (1989)
(Progress in mathematics ; Vol. 76)

NE: GT

Printed on acid-free paper.

ISBN-13: 978-1-4612-8216-7 e-ISBN-13: 978-1-4612-3696-2
DOI: 10.1007/978-1-4612-3696-2

© Birkhäuser Boston, 1989

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without prior permission of the copyright owner.

Text prepared by the authors in camera-ready form.

9 8 7 6 5 4 3 2 1

Preface

This is the first of two volumes representing the current state of knowledge about Enriques surfaces which occupy one of the classes in the classification of algebraic surfaces. Recent improvements in our understanding of algebraic surfaces over fields of positive characteristic allowed us to approach the subject from a completely geometric point of view although heavily relying on algebraic methods.

Some of the techniques presented in this book can be applied to the study of algebraic surfaces of other types. We hope that it will make this book of particular interest to a wider range of research mathematicians and graduate students.

Acknowledgements. The undertaking of this project was made possible by the support of several institutions. Our mutual cooperation began at the University of Warwick and the Max Planck Institute of Mathematics in 1982/83. Most of the work in this volume was done during the visit of the first author at the University of Michigan in 1984-1986. The second author was supported during all these years by grants from the National Science Foundation.

During the course of this work, many helpful discussions were held with various mathematicians. We acknowledge our special gratitude to Wolf Barth, Bill Lang, Eduard Looijenga, Chris Peters, and Miles Reid.

Contents

Introduction.....	1
Chapter 0. Preliminaries	
S1. Double covers.....	9
S2. Rational double points.....	24
S3. Del Pezzo surfaces.....	34
S4. Symmetric quartic Del Pezzo surfaces.....	39
S5. Symmetric cubic Del Pezzo surfaces.....	57
S6. Prym canonical maps.....	62
S7. The Picard scheme.....	65
Bibliographical notes.....	71
Chapter 1. Enriques surfaces: generalities	
S1. Classification of algebraic surfaces.....	72
S2. The Picard group.....	75
S3. The $K3$ -cover.....	83
S4. Differential invariants.....	88
S5. Riemann-Roch and a vanishing theorem.....	95
S6. Examples.....	99
Bibliographical notes.....	101
Chapter 11. Lattices and root bases	
S1. Generalities.....	103
S2. Root bases and their Weyl groups.....	106
S3. Root bases of finite and affine type.....	110
S4. Root bases of hyperbolic type.....	112
S5. The Enriques lattice.....	117
S6. The Reye lattice.....	128

§7. The function φ_M	134
§8. 2-congruence subgroups of finite Weyl groups.....	140
§9. The factor group $W/W(2)$	145
§10. The structure of $W(2)$	150
Bibliographical notes.....	162
Tables.....	163
Chapter III. The geometry of the Enriques lattice.	
§1. Divisors of canonical type.....	166
§2. The nodal chamber.....	175
§3. Canonical r -sequences and $U_{[r]}$ -markings.....	178
§4. U -markings.....	182
§5. $U_{[3]}$ -markings.....	186
§6. Linear systems $ C $ with $C^2 \leq 10$	221
Bibliographical notes.....	224
Chapter IV. Projective models.	
§1. Preliminaries.....	226
§2. Linear systems on K3-surfaces.....	229
§3. Numerical connectedness.....	232
§4. Base-points.....	240
§5. Hyperelliptic maps.....	243
§6. Birational maps.....	249
§7. Superelliptic maps.....	253
§8. The branch locus of superelliptic maps.....	260
§9. Projective models of degree ≤ 10	268
§10. Applications to linear systems.....	279
Appendix. A theorem of Igor Reider.....	281
Bibliographical notes.....	284
Chapter V. Genus 1 fibrations.	
§1. Genus 1 fibrations: generalities.....	285
§2. The Picard group.....	292
§3. Jacobian fibrations.....	302
§4. Ogg-Shafarevich theory.....	312

S5. Weierstrass models.....	326
S6. Genus 1 fibrations on rational surfaces.....	345
S7. Genus 1 fibrations on Enriques surfaces.....	363
Bibliographical notes.....	372
Bibliography	376
Index	389
Glossary of notations	394