Arithmetic Geometry

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Preface

This volume is the result of a (mainly) instructional conference on arithmetic geometry, held from July 30 through August 10, 1984 at the University of Connecticut in Storrs. This volume contains expanded versions of almost all the instructional lectures given during the conference. In addition to these expository lectures, this volume contains a translation into English of Faltings' seminal paper which provided the inspiration for the conference. We thank Professor Faltings for his permission to publish the translation and Edward Shipz who did the translation.

We thank all the people who spoke at the Storrs conference, both for helping to make it a successful meeting and enabling us to publish this volume. We would especially like to thank David Rohrlich, who delivered the lectures on height functions (Chapter VI) when the second editor was unavoidably detained. In addition to the editors, Michael Artin and John Tate served on the organizing committee for the conference and much of the success of the conference was due to them—our thanks go to them for their assistance.

Finally, the conference was only made possible through generous grants from the Vaughn Foundation and the National Science Foundation.

December, 1985

G. Cornell
J. H. Silverman

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Introduction

The chapters of this book, with the exception of Chapters II, XI and XII, are expanded versions of the lectures given at the Storrs Conference. They are intended, as was the conference, to introduce many of the ideas and techniques currently being used in arithmetic geometry; and in particular to explicate the tools used by Faltings in his proof of the Isogeny, Shafarevich and Mordell conjectures.

The first chapter is a brief overview, by Faltings himself, of the history leading up to the proof of the Mordell conjecture, and the second is a translation from the German of Faltings' paper in which he proved all three conjectures. The heart of this book, Chapters III through IX, contain (with varying amounts of detail) all of the results used in Faltings' paper. In particular, there is a thorough treatment of finite group schemes and p-divisible groups (Chapter III), Abelian and Jacobian varieties and schemes (Chapters IV, V, VII and VIII), their moduli spaces (Chapter IX) and height functions (Chapter VI). The prerequisites vary for each chapter, but in general, little is needed beyond what would normally be covered in one-year graduate courses in algebraic number theory and algebraic geometry.

After a brief chapter to illustrate the general theory for the particular case of elliptic curves (Chapter X), there are four chapters devoted to the theory of local height functions and arithmetic (Arakelov) intersection theory. Finally, Chapter XV contains an exposition of Vojta's far-reaching conjecture, whose consequences would include many of the standard finiteness theorems and outstanding conjectures in arithmetic geometry.

The editors hope that this volume will provide a path into the forest that is modern arithmetic geometry, wherein you will discover the beautiful flowers that blossom when arithmetic and geometry are intertwined, and there perchance, discover some new, exotic species heretofore unknown to the world of mathematics.