The Mathematics of Sonya Kovalevskaya

ИМПЕРАТОРСКАЯ АКАДЕМІЯ НАУКЪ

ПА ОСНОВАНІИ ВЫСОЧАЙШЕ ДАРОВАННАГО ЕЙ ВЪ 8-Й ДЕНЬ ЯНВАРЯ 1836 ГОДА

ИЗБРАЛА

СОФЬЮ ВАСИЛЬЕВНУ КОВАЛЕВСКУЮ

СОСТОЯЩУЮ ПРОФЕСОРОМЪ СТОКГОЛЬМСКАГО УНИВЕРСИТЕТА

ВЪ СВОИ ЧЛЕНЫ-КОРЕСПОНДЕНТЫ ПО РАЗРЯДУ МАТЕМАТИЧЕСКИХЪ НАУКЪ

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ВИЦЕ-ПРЕЗИДЕНТЪ

НЕПРЕМѢННЫЙ СЕКРЕТАРЬ

Kovalevskaya's certificate of election as a corresponding member of the Imperial Academy of Sciences, dated 29 December 1889 (10 January 1890). The certificate reads: The Imperial Academy of Sciences, by the authority of its supreme mandate of 8 January 1836, has elected Sophia Vasilievna Kovalevskaya, professor of the University of Stockholm, a corresponding member in the section of mathematical sciences. (Courtesy of the Institut Mittag-Leffler.)

Roger Cooke

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With 21 Figures

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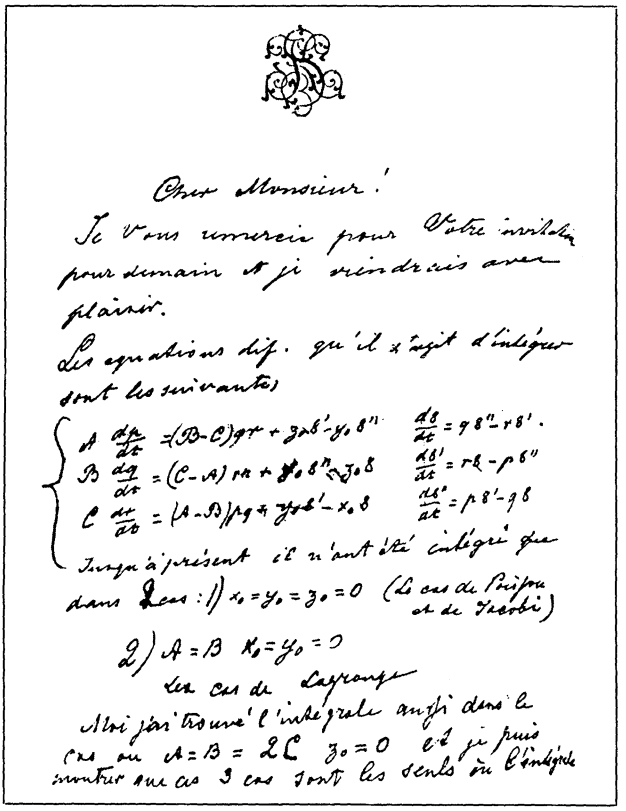
To Cathie, who keeps me in touch with what is most important,
To Katya, Andrew, and Hilary, who bring us joy every day,
To my father, who gave me an education beyond books and classrooms,
and
In loving memory of my mother



Letter from Kovalevskaya communicating her discovery of the Kovalevskaya case of a rotating rigid body. The letter was in Mittag-Leffler's possession and is possibly to him, though she usually addressed him less formally at the time the letter was written. (Courtesy of the Institut Mittag-Leffler.)

Preface

This book is the result of a decision taken in 1980 to begin studying the history of mathematics in the nineteenth century. I hoped by doing it to learn something of value about Kovalevskaya herself and about the mathematical world she inhabited. Having been trained as a mathematician, I also hoped to learn something about the proper approach to the history of the subject. The decision to begin the study with Kovalevskaya, apart from the intrinsic interest of Kovalevskaya herself, was primarily based upon the fact that the writing on her in English had been done by people who were interested in sociological and psychological aspects of her life. None of these writings discussed her mathematical work in much detail. This omission seemed to me a serious one in biographical studies of a woman whose primary significance was her mathematical work. In regard to both the content of nineteenth century mathematics and the nature of the history of mathematics I learned a great deal from writing this book. The attempt to put Kovalevskaya's work in historical context involved reading dozens of significant papers by great mathematicians. In many cases, I fear, the purport of these papers is better known to many of my readers than to me. If I persevered despite misgivings, my excuse is that this book is, after all, primarily about Kovalevskaya. If specialists in Euler, Cauchy, etc., find omissions or misinterpretations in my handling of the works of these authors, I can only plead that the background of a painting never has the same clarity as the foreground. In reading secondary sources I found noticeable differences between the accounts written by mathematicians and those written by historians. Among other differences, the former devote much more space to the technical details of how a result was obtained, while the latter tend to emphasize the influence of one idea or school upon another. As a newcomer to the field, I considered it my place to learn as much as I could from both mathematicians and historians. Naturally, the present work is written in the "mathematician's" style, though I have tried to make it as accessible as possible to nonspecialists.

Since Kovalevskaya's periods of mathematical activity, first as a student of Weierstrass and later as a mature scholar, were separated by a 6-year period of more or less domestic life, this natural division has been used as the plan for this book. In order to fix the chronology, the chapters discussing her work during these two periods are preceded by chapters of biography. It is very light biography, written to entertain as well as to instruct. It skims lightly over the years, touching down at points that seemed particularly interesting. The reader should by no means infer that the incidents reported here for any given year were necessarily the most important ones for understanding Kovalevskaya's life. For a serious scientific biography of Kovalevskaya the reader should turn to the excellent recent account by Ann Hibner Koblitz

(1983). Readers who know Russian will also find much of interest in the book of P. Ya. Kochina (1981). The present book uses the two just mentioned extensively as sources. The primary sources used in Chapters 1, 5, and 9 of the present book are in the archives of the Institut Mittag-Leffler. All letters quoted in this book, unless otherwise cited, come from that source.

The motivation for this book resides in the six chapters of mathematical analysis, in which an attempt is made to place Kovalevskaya's work in context within the history of nineteenth-century analysis. The effect of focusing on, and, so to speak, enlarging Kovalevskaya's papers, is a certain distortion of the picture. To make this book of reasonable length and readability, it was necessary to restrict the discussion of the works of her predecessors to the most significant results relevant to Kovalevskaya's work. Such a restriction makes the history of each topic resemble a ladder on which each successive mathematician moved one rung higher than his predecessors, whereas the reality is more like a tree, with branches sprouting in all directions. With that caveat I trust the reader will be better able to place Kovalevskaya's work in perspective after reading this book than before.

A few words are in order about the text itself, especially the mathematical analysis in Chapters 2–4 and 6–8. These chapters contain too few formulas to constitute a textbook exposition of the subjects they discuss and too many for a popular exposition. My impression after reading older histories of mathematics which discuss the works of great mathematicians in prose has been that such a style conveys no real idea of what the mathematics was. The formulas seem essential to any real communication of ideas. They should be used however as one would use a child's model airplane to explain that invention to someone who had never seen one. One would not expect to fly in a model airplane. For the same reason, one should not take the formulas in this book, even the long strings of formulas, for an attempt at a mathematical proof. The formulas are in the book only as a means of referring to the mathematics.

The six appendices were written for two reasons. The first was to make the book more "user-friendly" to a hypothetical undergraduate reader who has had only a year or two of mathematics. With these appendices, such a reader should be able to understand the gist of what is said in the text. The second reason for the appendices is that in the text it was necessary to touch on two topics (Weierstrass' method of solving partial differential equations of a certain form, and Jacobi's last multiplier method) which are no longer part of the basic courses in differential equations. I thought that mathematicians who, like me, had not seen these topics before, would be interested in a more detailed exposition of them.

Finally I should also explain my system, or rather my lack of system, in transliterating Cyrillic. Since the ending -sky is familiar to Europeans and Americans in such names as Dostoevsky and Tchaikovsky, I see no point in using -skii. In general Russian words are transliterated in a way which will cause the average English-speaking reader to pronounce them in an approximately correct manner. Those who know Russian will recognize these words

anyway, and those who do not will suffer no harm from reading a nonstandard transliteration. Knowing that Kovalevskaya's maternal ancestors came from Germany, I refer to them by the name Schubert, rather than Shubert. On the other hand, I have retained the spelling Mendel'son in preference to Mendelssohn, because the woman in question was Polish. In connection with Russian names, many conversations with non-Russian-speaking devotees of Russian literature have convinced me that the invariable sequence of given-name-plus-patronymic is excessively wearisome to most readers. Therefore I make very little use of patronymics, trusting that any Russian-speaking readers will understand that no disrespect is intended. Similarly, in order to avoid confusing my readers with the many diminutives of the name Sophia I consistently use "Sonya" even though Kovalevskaya did not get that nickname until adulthood.

It is a pleasure to record my gratitude to the many organizations and individuals who have assisted me in carrying out this project. My sincere thanks go

to Patricia and Philip Kitcher for many years of friendship and for advice on both technical and scholarly problems;

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to Ivor Grattan-Guinness for invaluable help; I trust he will accept this acknowledgment in lieu of a citation at each point where I have used one of the references he supplied; he will know from comparing this book with the draft he read how many improvements are due to his advice;

to the London Mathematical Society, the British Society for the History of Mathematics, University College London, and the British Museum for access to sources and other assistance during my year in England. Indeed I would like to thank the entire United Kingdom for making the year 1981–82 so pleasant for me and my family;

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Roger Cooke

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