

# **Applied Mathematical Sciences**

Volume 70

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(continued following index)

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**Integral Manifolds  
and Inertial Manifolds for  
Dissipative Partial  
Differential Equations**



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Mathematics Subject Classification (1980): 58GXX

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Library of Congress Cataloging-in-Publication Data

Integral manifolds and inertial manifolds for dissipative partial  
differential equations / P. Constantin ... [et al.].

p. cm. — (Applied mathematical sciences; v. 70)

Bibliography: p.

Includes index.

ISBN-13:978-1-4612-8131-3

1. Differential equations, Partial. 2. Manifolds (Mathematics)

I. Constantin, P. (Peter), 1951–. II. Series: Applied  
mathematical sciences (Springer-Verlag New York Inc.); v. 70.

QA1.A647 vol. 70

[QA377]

510 s—dc19

[515.3'53]

88-20021

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Softcover reprint of the hardcover 1st edition 1989

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Typeset by Asco Trade Typesetting Ltd., Hong Kong.

9 8 7 6 5 4 3 2

ISBN-13:978-1-4612-8131-3

e-ISBN-13:978-1-4612-3506-4

DOI: 10.1007/978-1-4612-3506-4

# Preface

This work was initiated in the summer of 1985 while all of the authors were at the Center of Nonlinear Studies of the Los Alamos National Laboratory; it was then continued and polished while the authors were at Indiana University, at the University of Paris-Sud (Orsay), and again at Los Alamos in 1986 and 1987.

Our aim was to present a direct geometric approach in the theory of inertial manifolds (global analogs of the unstable-center manifolds) for dissipative partial differential equations. This approach, based on Cauchy integral manifolds for which the solutions of the partial differential equations are the generating characteristic curves, has the advantage that it provides a sound basis for numerical Galerkin schemes obtained by approximating the inertial manifold.

The work is self-contained and the prerequisites are at the level of a graduate student. The theoretical part of the work is developed in Chapters 2–14, while in Chapters 15–19 we apply the theory to several remarkable partial differential equations.

We wish to thank G. R. Sell, J. M. Ghidaglia, M. Jolly, J. C. Saut, and E. Titi for their interest and remarks. We are also indebted to Fred Flowers for the careful typing of the manuscript.

# Acknowledgments

This research was partially supported by the Applied Mathematical Sciences Program of the U.S. Department of Energy, Contract DE-ACd02-82ER12049 and Grant DE-FG02-86ER25020; by the National Science Foundation, Grant NSF-DMS-8602031, and the Research Fund of Indiana University; and by the Center for Nonlinear Studies, Los Alamos National Laboratory, operated by the University of California under Contract W-7405-ENG-36. P. Constantin acknowledges a Sloan research fellowship.

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