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Computational Methods in Optimal Control Theory and Practice



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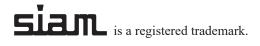
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Contents

List of Figures									
List	List of Tables xi								
Prefa	ace		xiii						
1	Introduction								
	1.1	Some History	1						
	1.2	Notation	6						
	1.3	Local versus Global Optima	8						
	1.4	Approaches and Discretizations	11						
	1.5	A Standard Problem Formulation	17						
2	Convergence Theory								
	2.1	The Abstract Setting	23						
	2.2	Contraction Mapping Theory	24						
	2.3	Picard Existence Theory	29						
	2.4	An Existence Result for Ordinary Equations	30						
	2.5	An Existence Result for Generalized Equations	32						
3	Truncation Errors in Runge-Kutta Methods								
	3.1	First-Order Optimality Conditions	35						
	3.2	Optimality Conditions for Runge–Kutta Discretizations	38						
	3.3	Truncation Error for Euler's Method	40						
	3.4	Truncation Error for Euler's Improved Method	43						
	3.5	Truncation Error for Symplectic Runge–Kutta Schemes	50						
	3.6	High-Order Symplectic Runge–Kutta Schemes	56						
	3.7	Condensed Controls with Second-Order Accuracy	60						
4	Convergence Theory for Runge-Kutta Methods								
	4.1	Convergence of Euler's Method	65						
	4.2	Convergence of Euler's Improved Method	76						
	4.3	Convergence of Symplectic Runge–Kutta Schemes	79						
	4.4	Convergence of Schemes with Condensed Controls	91						
5	Orthogonal Collocation								
	5.1	Polynomial Approximations	95						
	5.2	Orthogonal Collocation	100						
	5.3	A Runge–Kutta View of <i>hp</i> -Orthogonal Collocation	104						

viii Contents

	5.4	Truncation	on Error for Orthogonal Collocation	110		
	5.5		ence of Orthogonal Collocation			
6	Endpo	oint Const	raints and Discontinuous Controls	123		
	6.1	Terminal	Constraints without Control	123		
	6.2	Free Terminal Time				
	6.3	Terminal Constraints with Control				
		6.3.1	Analysis of Perturbed Boundary-Value Problem .	135		
		6.3.2	Objective Derivative Relative to a Switch Point			
	6.4	Example	141			
		6.4.1	Harmonic Oscillator Minimum Time Problem			
		6.4.2	Singular Control Problems	143		
		6.4.3	Generalized Problems			
A	Minim	num Princ	iple	151		
Bibl	iograph	y		155		
Index						

List of Figures

A mass sliding on a frictionless surface while attached to a wall by a					
spring					
A plot of p_2 for $\theta = 0$					
Plot of the switch curve and a minimum time trajectory					
Plot of solution to obstacle problem 6					
Plot of the optimal altitude (km) of the plane versus time (s) 10					
Plot of the optimal Mach number of the plane versus height (km) 10					
Plot of the optimal α of the plane versus time (s)					
Time mesh in the discretization					
The solution of the model problem (1.16)					
$\log_{10} E_N$ versus $\log_{10} h$, where E_N = is the maximum discrete state					
error at the mesh points					
The normal cone when \mathbf{u} lies at the vertex of a cone \mathscr{U} and when \mathscr{U} is a sphere with \mathbf{u} on its boundary					
Plot of the optimal state and control for problem (3.47)					
Plot of the zeros of P_{14} , the Legendre polynomial of degree 14 96					
Plot of the flipped Radau points for $N=14$, where $\sigma_{14}=1$ and					
$\sigma_1 > -1$					
Euler approximation to the optimal control for harmonic oscillator 143					
Trajectory of minimum time harmonic oscillator					
Euler approximation to solution of catalyst mixing problem, $N=100$ 145					
Euler approximation to solution of catalyst mixing problem, $N=1000.$. 146					
Euler approximation to catalyst mixing problem for $N=100$ and dif-					
ferent choices of the penalty parameter					