

# Applied Mathematical Sciences

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## MANUSCRIPTS

The Editors welcome all inquiries regarding the submission of manuscripts for the series. Final preparation of all manuscripts will take place in the editorial offices of the series in the Division of Applied Mathematics, Brown University, Providence, Rhode Island.

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SPRINGER-VERLAG NEW YORK INC., 175 Fifth Avenue, New York, N. Y. 10010

*Printed in U.S.A.*

# **Applied Mathematical Sciences | Volume 17**

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# Optimization Problems

Translated by:  
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Springer-Verlag New York • Heidelberg • Berlin  
1975

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AMS Classifications  
Primary: 49-01  
Secondary: 65Fxx, 90C05, 90D40

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*Library of Congress Cataloging in Publication Data*

Collatz, Lothar, 1910–  
Optimization problems.

(Applied mathematical sciences, v. 17)

Translation of Optimierungsaufgaben.

Bibliography: p.

Includes index.

1. Mathematical optimization. 2. Game theory.

1. Wetterling, Wolfgang W. E., 1932– joint author.

II. Title. III. Series.

QA1.A647 vol. 17 [QA402.5] 510'.8s [519.7]

75-15795

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Softcover reprint of the hardcover 1st Edition 1975

ISBN-13: 978-0-387-90143-5

e-ISBN-13: 978-1-4612-6378-4

DOI:10.1007/978-1-4612-6378-4

## PREFACE

The German edition of this book, first published in 1966, has been quite popular; we did not, however, consider publishing an English edition because a number of excellent textbooks in this field already exist. In recent years, however, the wish was frequently expressed that, especially, the description of the relationships between optimization and other subfields of mathematics, which is not to be found in this form in other texts, might be made available to a wider readership; so it was with this in mind that, belatedly, a translation was undertaken after all.

Since the appearance of the German edition, the field of optimization has continued to develop at an unabated rate. A completely current presentation would have required a total reworking of the book; unfortunately, this was not possible. For example, we had to ignore the extensive progress which has been made in the development of numerical methods which do not require convexity assumptions to find local maxima and minima of non-linear optimization problems. These methods are also applicable to boundary value, and other, problems. Many new results, both of a numerical and a theoretical nature, which are especially relevant to applications, are to be found in the areas of optimal control and integer optimization.

Although these and many other new developments had to be ignored, we hope that the book continues to satisfy the goals set forth in the preface to the German edition.

Finally, we want to take this opportunity to express our gratitude, to Peter R. Wadsack for a careful translation, and to Springer Verlag for kind cooperation.

## FROM THE PREFACE TO THE GERMAN EDITION

With this book we would like to provide an introduction to a field which has developed into a great new branch of knowledge in the last thirty years. Indeed, it continues to be the object of intensive mathematical research. This rapid development has been possible because there exists a particularly close contact between theory and application.

Optimization problems have appeared in very different applied fields, including such fields as political economics and management science, for example, where little use was formerly made of mathematical methods. It also has become apparent that questions from very different areas of numerical mathematics may be regarded as examples of optimization. Thus, many types of initial value and boundary value problems of ordinary and partial differential equations, as well as approximation problems, game theoretic questions, and others, reduce to optimization problems. As this field has grown in importance, the number of texts has increased. Thus some justification for yet another text might be required. Now most existing texts deal with some subfield, whether linear or non-linear optimization, game theory, or whatever. So it became our intention to provide a certain overview of the entire field with this book, while emphasizing the connections and interrelations among different fields and subfields, including those previously mentioned. Since it is also our impression that these new fields -- for example, the beautiful general theorems on systems of equations and in-

equalities -- are not yet generally known, even in mathematical circles, we want to use this book to provide a general, easily comprehensible, and for the practitioner, readily accessible, introduction to this varied field, complete with proofs and unobscured by excessive computational detail. Thus, several deeper concepts, such as the theory of optimal processes (due to Pontrjagin), for one example, or the theory of dynamic optimization (due to Bellman), for another, are not discussed.

The book resulted from a number of courses in the subject given by the authors at the Universität Hamburg. In addition, one of the authors included the theorems of the alternative for systems of equations and inequalities, up to the duality theorem of linear optimization (§5 of this book) in an introductory course on "Analytic Geometry and Algebra"; for these theorems may be presented in a few hours as an immediate sequel to matrix theory and the concept of linear independence of vectors. It seems desirable that the young student become familiar with these things. In some countries they already are covered in high school seminars, for which they are well suited. They contribute to the dissemination of mathematics into other sciences and thus their significance will certainly grow in the future.



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