Undergraduate Texts in Mathematics

Kai Lai Chung

Elementary Probability Theory with Stochastic Processes

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'Tis the good reader that makes the good book.
Ralph Waldo Emerson

Preface to the Third Edition

A new feature of this edition consists of photographs of eight masters in the contemporary development of probability theory. All of them appear in the body of the book, though the few references there merely serve to give a glimpse of their manifold contributions. It is hoped that these vivid pictures will inspire in the reader a feeling that our science is a live endeavor created and pursued by real personalities. I have had the privilege of meeting and knowing most of them after studying their works and now take pleasure in introducing them to a younger generation. In collecting the photographs I had the kind assistance of Drs Marie-Hélène Schwartz, Joanne Elliot, Milo Keynes and Yu. A. Rozanov, to whom warm thanks are due.

A German edition of the book has just been published. I am most grateful to Dr. Herbert Vogt for his careful translation which resulted also in a considerable number of improvements on the text of this edition. Other readers who were kind enough to send their comments include Marvin Greenberg, Louise Hay, Nora Holmquist, H.-E. Lahmann, and Fred Wolock. Springer-Verlag is to be complimented once again for its willingness to make its books "immer besser."

K.L.C.
September 19, 1978

Preface to the Second Edition

A determined effort was made to correct the errors in the first edition. This task was assisted by: Chao Hung-po, J. L. Doob, R. M. Exner, W. H. Fleming, A. M. Gleason, Karen Kafador, S. H. Polit, and P. van Moerbeke. Miss Kafador and Dr. Polit compiled particularly careful lists of suggestions. The most distressing errors were in the Solutions to Problems. All of them have now been checked by myself from Chapter 1 to 5, and by Mr. Chao from Chapter 6 to 8. It is my fervent hope that few remnant mistakes remain in that sector. A few small improvements and additions were also made, but not all advice can be heeded at this juncture. Users of the book are implored to send in any criticism and commentary, to be taken into consideration in a future edition. Thanks are due to the staff of Springer-Verlag for making this revision possible so soon after the publication of the book.

K. L. C.

Preface to the First Edition

In the past half-century the theory of probability has grown from a minor isolated theme into a broad and intensive discipline interacting with many other branches of mathematics. At the same time it is playing a central role in the mathematization of various applied sciences such as statistics, operations research, biology, economics and psychology—to name a few to which the prefix "mathematical" has so far been firmly attached. The coming-of-age of probability has been reflected in the change of contents of textbooks on the subject. In the old days most of these books showed a visible split-personality torn between the combinatorial games of chance and the so-called "theory of errors" centering in the normal distribution. This period ended with the appearance of Feller's classic treatise (see [Feller 1]†) in 1950, from the manuscript of which I gave my first substantial course in probability. With the passage of time probability theory and its applications have won a place in the college curriculum as a mathematical discipline essential to many fields of study. The elements of the theory are now given at different levels, sometimes even before calculus. The present textbook is intended for a course at about the sophomore level. It presupposes no prior acquaintance with the subject and the first three chapters can be read largely without the benefit of calculus. The next three chapters require a working knowledge of infinite series and related topics, and for the discussion involving random variables with densities some calculus is of course assumed. These parts dealing with the "continuous case" as distinguished from the "discrete case" are easily separated and may be postponed. The contents of the first six chapters should form the backbone of any meaningful first introduction to probability theory. Thereafter a reasonable selection includes: §7.1 (Poisson distribution, which may be inserted earlier in the course), some kind of going over of §7.3, 7.4, 7.6 (normal distribution and the law of large numbers), and §8.1 (simple random walks which are both stimulating and useful). All this can be covered in a semester but for a quarter system some abridgment will be necessary. Specifically, for such a short course Chapters 1 and 3 may be skimmed through and the asterisked material omitted. In any case a solid treatment of the normal approximation theorem in Chapter 7 should be attempted only if time is available as in a semester or two-quarter course. The final Chapter 8 gives a self-contained elementary account of Markov chains and is an extension of the main course at a somewhat more mature level. Together with the asterisked sections 5.3, 5.4 (sequential sampling and Pólya urn scheme) and 7.2 (Poisson process), and perhaps some filling in from the Appendices, the material provides a gradual and concrete passage into the domain of sto-

† Names in square brackets refer to the list of General References on p. 307.

chastic processes. With these topics included the book will be suitable for a two-quarter course, of the kind that I have repeatedly given to students of mathematical sciences and engineering. However, after the preparation of the first six chapters the reader may proceed to more specialized topics treated e.g. in the above mentioned treatise by Feller. If the reader has the adequate mathematical background, he will also be prepared to take a formal rigorous course such as presented in my own more advanced book [Chung 1].

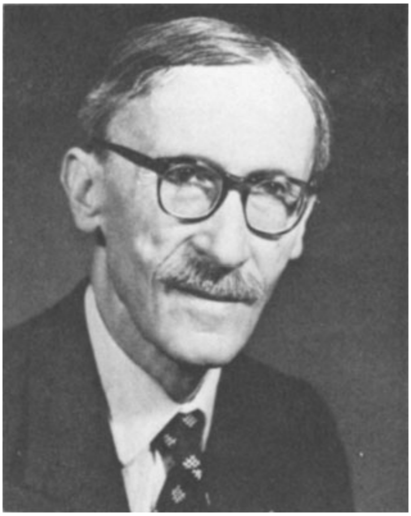
Much thought has gone into the selection, organization and presentation of the material to adapt it to classroom uses, but I have not tried to offer a slick package to fit in with an exact schedule or program such as popularly demanded at the quick-service counters. A certain amount of flexibility and choice is left to the instructor who can best judge what is right for his class. Each chapter contains some easy reading at the beginning, for motivation and illustration, so that the instructor may concentrate on the more formal aspects of the text. Each chapter also contains some slightly more challenging topics (e.g., §1.4, 2.5) for optional sampling. They are not meant to deter the beginner but to serve as an invitation to further study. The prevailing emphasis is on the thorough and deliberate discussion of the basic concepts and techniques of elementary probability theory with few frills and minimal technical complications. Many examples are chosen to anticipate the beginners' difficulties and to provoke better thinking. Often this is done by posing and answering some leading questions. Historical, philosophical and personal comments are inserted to add flavor to this lively subject. It is my hope that the reader will not only learn something from the book but may also derive a measure of enjoyment in so doing.

There are over two hundred exercises for the first six chapters and some eighty more for the last two. Many are easy, the harder ones indicated by asterisks, and all answers gathered at the end of the book. Asterisked sections and paragraphs deal with more special or elaborate material and may be skipped, but a little browsing in them is recommended.

The author of any elementary textbook owes of course a large debt to innumerable predecessors. More personal indebtedness is acknowledged below. Michel Nadzela wrote up a set of notes for a course I gave at Stanford in 1970. Gian-Carlo Rota, upon seeing these notes, gave me an early impetus toward transforming them into a book. D. G. Kendall commented on the first draft of several chapters and lent further moral support. J. L. Doob volunteered to read through most of the manuscript and offered many helpful suggestions. K. B. Erickson used some of the material in a course he taught. A. A. Balkema checked the almost final version and made numerous improvements. Dan Rudolph read the proofs together with me. Perfecto Mary drew those delightful pictures. Gail Lemmond did the typing with her usual efficiency and dependability. Finally, it is a pleasure to thank my old publisher Springer-Verlag for taking my new book to begin a new series of undergraduate texts.

K. L. C.
March 1974.

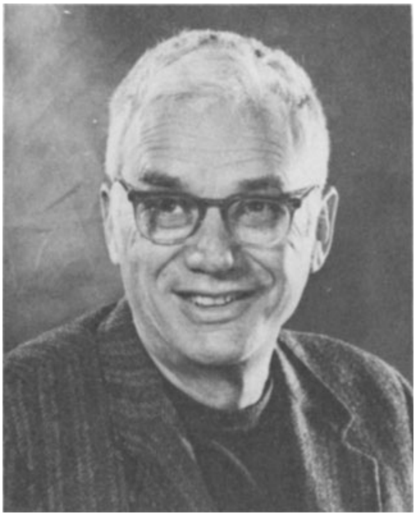
Borel



Lévy

Keynes

Feller



Doob

Pólya

Kolmogorov

Cramér

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