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Methods of Bifurcation Theory

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To Marie and Hazel

Preface

An alternative title for this book would perhaps be Nonlinear Analysis, Bifurcation Theory and Differential Equations. Our primary objective is to discuss those aspects of bifurcation theory which are particularly meaningful to differential equations.

To accomplish this objective and to make the book accessible to a wider audience, we have presented in detail much of the relevant background material from nonlinear functional analysis and the qualitative theory of differential equations. Since there is no good reference for some of the material, its inclusion seemed necessary.

Two distinct aspects of bifurcation theory are discussed—static and dynamic. Static bifurcation theory is concerned with the changes that occur in the structure of the set of zeros of a function as parameters in the function are varied. If the function is a gradient, then variational techniques play an important role and can be employed effectively even for global problems. If the function is not a gradient or if more detailed information is desired, the general theory is usually local. At the same time, the theory is constructive and valid when several independent parameters appear in the function. In differential equations, the equilibrium solutions are the zeros of the vector field. Therefore, methods in static bifurcation theory are directly applicable.

Dynamic bifurcation theory is concerned with the changes that occur in the structure of the limit sets of solutions of differential equations as parameters in the vector field are varied. For example, in addition to discussing the way that the set of zeros of the vector field (the equilibrium solutions) change through the static theory, the stability properties of these solutions must be considered. In fact, there is an intimate relationship between changes of stability and bifurcation. The dynamics in a differential equation can also introduce other types of bifurcations; for example, periodic orbits, homoclinic orbits, invariant tori. This introduces several difficulties which require rather advanced topics from differential equations for their resolution.

The introductory chapter is designed to acquaint the reader with some of the types of problems that occur in bifurcation theory. The tools from nonlinear functional analysis are presented in Chapter 2. Some of this material is used more extensively in the text than others, but all topics are a necessary part of the vocabulary of persons working in bifurcation theory.

Some of the presentations and details of proofs are different from standard ones.

Chapter 3 gives applications of the Implicit Function Theorem. These are not bifurcation problems. Some of the applications were chosen because the material is needed in later chapters. Others give good illustrations of some of the tools in Chapter 2.

Chapters 4–8 deal with static bifurcation theory. Chapter 4 contains the fundamental elements of variational theory together with serious applications to Hamiltonian systems, elliptic and hyperbolic problems.

Chapters 5–8 deal almost entirely with analytic methods in local static bifurcation theory. In Chapter 5, for functions depending on a scalar parameter, conditions are given to ensure that there is always a bifurcation near equilibrium. These conditions are based on the linear approximation and are independent of the nonlinearities. Some global results are also included.

In Chapter 6, the case of a one-dimensional null space for the linear approximation is analyzed in detail under generic conditions on the quadratic and cubic terms. The effects of symmetry are also discussed. Chapter 7 is concerned with the case where the linear approximation has a two-dimensional null space with the quadratic and cubic terms satisfying some nondegeneracy conditions. Both of these chapters contain constructive procedures in the analysis. Chapter 8 contains applications to the buckling of plates, chemical reactions and Duffing's equation.

Chapters 9–13 are devoted to dynamic bifurcation theory. Chapter 9 is concerned with the bifurcation from an equilibrium point in the case when the linear approximation has either one zero eigenvalue or a pair of purely imaginary eigenvalues. It is shown that all relevant information on existence and stability is contained in the bifurcation function obtained via the alternative method or the method Liapunov–Schmidt. The hypotheses on the linear part are the typical situation for one parameter families of vector fields. Chapter 10 is devoted to the other bifurcation phenomena that occur in the plane for typical one parameter families of autonomous vector fields.

In Chapter 11, we discuss periodic planar vector fields and especially Hamiltonian systems with a small damping and small periodic forcing term. Emphasis is placed on the existence of subharmonic solutions and the role of successive bifurcations through subharmonics in the creation of homoclinic points and a type of random behavior.

In Chapter 12, averaging, the theory of normal forms and the theory of integral manifolds for ordinary differential equations are presented. This material is relevant to the discussion of bifurcation to tori considered in that chapter as well as the problems in Chapter 13, which is devoted to the behavior of the solutions of a differential equation near an equilibrium point when the linear part of the vector field is typical of two parameter problems.

The topics in Chapter 14 on perturbation of the spectrum of linear operators is distinct from the ones in the previous chapters. It is included because the same methods can be applied to yield elementary proofs of some results in this field.

The material in this book can be easily adapted to several types of one semester courses. For example, four possible reasonable arrangements could be:

I. Chapters 1, 5, 6, 7, 8 with Sections 2.3–2.8 from Chapter 2.
II. Chapter 2, 3, 4.
III. Chapters 1, 9, 10 with Sections 2.3, 2.4, 2.5 from Chapter 2.
IV. Chapters 11, 12, 13.

Examples I, II, III are independent and require minimal knowledge of differential equations. Example IV can only be taught after III and requires more sophisticated concepts from differential equations.

The authors are indebted to numerous colleagues and students for their assistance in this work. We especially thank John Mallet–Paret with whom we had so many stimulating conversations about technique and method of presentation. Luis Magalhães also was of great assistance, especially in the presentation in Chapter 11 and the examples in Chapter 12. We have also been assisted by many persons in the preparation of the final manuscript. We are indebted especially to Eleanor Addison, Dorothy Libutti, Sandra Spinacci, Kate MacDougall, Mary Reynolds and Diane Norton. The second author is also indebted to the Guggenheim Foundation for a Fellowship during 1979–80.

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