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(continued following index)

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Vorticity and Turbulence

With 45 Illustrations



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Preface

This book provides an introduction to the theory of turbulence in fluids based on the representation of the flow by means of its vorticity field. It has long been understood that, at least in the case of incompressible flow, the vorticity representation is natural and physically transparent, yet the development of a theory of turbulence in this representation has been slow. The pioneering work of Onsager and of Joyce and Montgomery on the statistical mechanics of two-dimensional vortex systems has only recently been put on a firm mathematical footing, and the three-dimensional theory remains in parts speculative and even controversial.

The first three chapters of the book contain a reasonably standard introduction to homogeneous turbulence (the simplest case); a quick review of fluid mechanics is followed by a summary of the appropriate Fourier theory (more detailed than is customary in fluid mechanics) and by a summary of Kolmogorov's theory of the inertial range, slanted so as to dovetail with later vortex-based arguments. The possibility that the inertial spectrum is an equilibrium spectrum is raised.

The remainder of the book presents the vortex dynamics of turbulence, with as little mathematical and physical baggage as is compatible with clarity. In Chapter 4, the Onsager and Joyce-Montgomery discoveries in the two-dimensional case are presented from a contemporary point of view, and more rigorous recent treatments are briefly surveyed. This is where the peculiarities of vortex statistics, in particular negative (trans-infinite) temperatures, first appear. Chapter 5 summarizes the fractal geometry of vortex stretching, and Chapter 6 provides a brief but self-contained introduction to the tools needed for further analysis, in particular polymer

statistics, percolation, and real-space renormalization. In Chapter 7, these tools are used to analyze a simple model of three-dimensional vortex statistics. The Kolmogorov theory is revisited; a rationale is provided for the effectiveness of some large-eddy approximations; and an instructive contrast is drawn between classical and superfluid turbulence.

Some practical information about approximation procedures is provided in the book, as well as tools for assessing the plausibility of approximation schemes. The emphasis, however, is on the understanding of turbulence—its origin, mechanics, spectra, organized structures, energy budget, and renormalization. The physical space methodology is natural, and makes the reasoning particularly straightforward. Open questions are indicated as such throughout the book.

February 1993

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