Applied Mathematical Sciences
Volume 103

Springer Science+Business Media, LLC

Applied Mathematical Sciences

1. *John:* Partial Differential Equations, 4th ed.
2. *Sirovich:* Techniques of Asymptotic Analysis.
3. *Hale:* Theory of Functional Differential Equations, 2nd ed.
4. *Percus:* Combinatorial Methods.
5. *von Mises/Friedrichs:* Fluid Dynamics.
6. *Freiberger/Grenander:* A Short Course in Computational Probability and Statistics.
7. *Pipkin:* Lectures on Viscoelasticity Theory.
8. *Giacoglia:* Perturbation Methods in Non-linear Systems.
9. *Friedrichs:* Spectral Theory of Operators in Hilbert Space.
10. *Stroud:* Numerical Quadrature and Solution of Ordinary Differential Equations.
11. *Wolovich:* Linear Multivariable Systems.
12. *Berkovitz:* Optimal Control Theory.
13. *Bluman/Cole:* Similarity Methods for Differential Equations.
14. *Yoshizawa:* Stability Theory and the Existence of Periodic Solution and Almost Periodic Solutions.
15. *Braun:* Differential Equations and Their Applications, 3rd ed.
16. *Lefschetz:* Applications of Algebraic Topology.
17. *Collatz/Wetterling:* Optimization Problems.
18. *Grenander:* Pattern Synthesis: Lectures in Pattern Theory, Vol. I.
19. *Marsden/McCracken:* Hopf Bifurcation and Its Applications.
20. *Driver:* Ordinary and Delay Differential Equations.
21. *Courant/Friedrichs:* Supersonic Flow and Shock Waves.
22. *Rouche/Habets/Laloy:* Stability Theory by Liapunov's Direct Method.
23. *Lamperti:* Stochastic Processes: A Survey of the Mathematical Theory.
24. *Grenander:* Pattern Analysis: Lectures in Pattern Theory, Vol. II.
25. *Davies:* Integral Transforms and Their Applications, 2nd ed.
26. *Kushner/Clark:* Stochastic Approximation Methods for Constrained and Unconstrained Systems.
27. *de Boor:* A Practical Guide to Splines.
28. *Keilson:* Markov Chain Models—Rarity and Exponentiality.
29. *de Veubeke:* A Course in Elasticity.
30. *Shiatycki:* Geometric Quantization and Quantum Mechanics.
31. *Reid:* Sturmian Theory for Ordinary Differential Equations.
32. *Meis/Markowitz:* Numerical Solution of Partial Differential Equations.
33. *Grenander:* Regular Structures: Lectures in Pattern Theory, Vol. III.
34. *Kevorkian/Cole:* Perturbation Methods in Applied Mathematics.
35. *Carr:* Applications of Centre Manifold Theory.
36. *Bengtsson/Ghil/Källén:* Dynamic Meteorology: Data Assimilation Methods.
37. *Saperstone:* Semidynamical Systems in Infinite Dimensional Spaces.
38. *Lichtenberg/Lieberman:* Regular and Chaotic Dynamics, 2nd ed.
39. *Piccini/Stampacchia/Vidossich:* Ordinary Differential Equations in $\mathbf{R}^n$.
40. *Naylor/Sell:* Linear Operator Theory in Engineering and Science.
41. *Sparrow:* The Lorenz Equations: Bifurcations, Chaos, and Strange Attractors.
42. *Guckenheimer/Holmes:* Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields.
43. *Ockendon/Taylor:* Inviscid Fluid Flows.
44. *Pazy:* Semigroups of Linear Operators and Applications to Partial Differential Equations.
45. *Glashoff/Gustafson:* Linear Operations and Approximation: An Introduction to the Theoretical Analysis and Numerical Treatment of Semi-Infinite Programs.
46. *Wilcox:* Scattering Theory for Diffraction Gratings.
47. *Hale et al:* An Introduction to Infinite Dimensional Dynamical Systems—Geometric Theory.
48. *Murray:* Asymptotic Analysis.
49. *Ladyzhenskaya:* The Boundary-Value Problems of Mathematical Physics.
50. *Wilcox:* Sound Propagation in Stratified Fluids.
51. *Golubitsky/Schaeffer:* Bifurcation and Groups in Bifurcation Theory, Vol. I.
52. *Chipot:* Variational Inequalities and Flow in Porous Media.
53. *Majda:* Compressible Fluid Flow and System of Conservation Laws in Several Space Variables.
54. *Wasow:* Linear Turning Point Theory.
55. *Yosida:* Operational Calculus: A Theory of Hyperfunctions.
56. *Chang/Howes:* Nonlinear Singular Perturbation Phenomena: Theory and Applications.
57. *Reinhardt:* Analysis of Approximation Methods for Differential and Integral Equations.
58. *Dwoyer/Hussaini/Voigt (eds):* Theoretical Approaches to Turbulence.
59. *Sanders/Verhulst:* Averaging Methods in Nonlinear Dynamical Systems.
60. *Ghil/Childress:* Topics in Geophysical Dynamics: Atmospheric Dynamics, Dynamo Theory and Climate Dynamics.

(continued following index)

Alexandre J. Chorin

Vorticity and Turbulence

With 45 Illustrations

Springer

Alexandre J. Chorin
Department of Mathematics
University of California
Berkeley, CA 94720
USA

Series Editors

J.E. Marsden
Control and Dynamical Systems 116-18
California Institute of Technology
Pasadena, CA 91125
USA

L. Sirovich
Division of Applied Mathematics
Brown University
Providence, RI 02912
USA

Mathematical Subject Classification (1991): 76Fxx, 60J65, 62M40

Library of Congress Cataloging-in-Publication Data
Chorin, Alexandre Joel.
Vorticity and Turbulence / Alexandre Chorin.
p. cm. – (Applied mathematical sciences : v. 103)
Includes bibliographical references and index.
ISBN 978-1-4612-6459-0 ISBN 978-1-4419-8728-0 (eBook)
DOI 10.1007/978-1-4419-8728-0

(Berlin : alk. paper)
1. Vortex-motion. 2. Turbulence. 3. Fluid mechanics. I. Title.
II. Series : Applied mathematical sciences (Springer-Verlag New York Inc.) : v. 103.
QC159.C48 1994
532′.0527–dc20 93-4311
Printed on acid-free paper.

Originally published by Springer-Verlag New York, Inc., in 1994
Softcover reprint of the hardcover 1st edition 1994

Production managed by Ken Dreyhaupt; manufacturing supervised by Genieve Shaw.
Photocomposed copy prepared from the author's AMS-LaTeX files.

9 8 7 6 5 4 3 2 (Corrected second printing, 1998)

Preface

This book provides an introduction to the theory of turbulence in fluids based on the representation of the flow by means of its vorticity field. It has long been understood that, at least in the case of incompressible flow, the vorticity representation is natural and physically transparent, yet the development of a theory of turbulence in this representation has been slow. The pioneering work of Onsager and of Joyce and Montgomery on the statistical mechanics of two-dimensional vortex systems has only recently been put on a firm mathematical footing, and the three-dimensional theory remains in parts speculative and even controversial.

The first three chapters of the book contain a reasonably standard introduction to homogeneous turbulence (the simplest case); a quick review of fluid mechanics is followed by a summary of the appropriate Fourier theory (more detailed than is customary in fluid mechanics) and by a summary of Kolmogorov's theory of the inertial range, slanted so as to dovetail with later vortex-based arguments. The possibility that the inertial spectrum is an equilibrium spectrum is raised.

The remainder of the book presents the vortex dynamics of turbulence, with as little mathematical and physical baggage as is compatible with clarity. In Chapter 4, the Onsager and Joyce-Montgomery discoveries in the two-dimensional case are presented from a contemporary point of view, and more rigorous recent treatments are briefly surveyed. This is where the peculiarities of vortex statistics, in particular negative (trans-infinite) temperatures, first appear. Chapter 5 summarizes the fractal geometry of vortex stretching, and Chapter 6 provides a brief but self-contained introduction to the tools needed for further analysis, in particular polymer statistics, percolation, and real-space renormalization. In Chapter 7, these tools are used to analyze a simple model of three-dimensional vortex statistics. The Kolmogorov theory is revisited; a rationale is provided for the effectiveness of some large-eddy approximations; and an instructive contrast is drawn between classical and superfluid turbulence.

Some practical information about approximation procedures is provided in the book, as well as tools for assessing the plausibility of approximation schemes. The emphasis, however, is on the understanding of turbulence—its origin, mechanics, spectra, organized structures, energy budget, and renormalization. The physical space methodology is natural, and makes the reasoning particularly straightforward. Open questions are indicated as such throughout the book.

February 1993

Contents