

Shiing-shen Chern

Complex Manifolds Without Potential Theory

(with an appendix on the geometry
of characteristic classes)

Second Edition

Revised Printing



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