

# Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann  
Subseries: Harvard/MIT  
Adviser: G. Sacks

521

---

Greg Cherlin

Model Theoretic Algebra  
Selected Topics

---



Springer-Verlag  
Berlin · Heidelberg · New York 1976

## Author

Greg Cherlin  
Department of Mathematics  
Rutgers University  
New Brunswick  
New Jersey 08903/USA

### Library of Congress Cataloging in Publication Data

Cherlin, Greg, 1948-

Model theoretic algebra.

(Lecture notes in mathematics ; 521)

Bibliography: p.

Includes indexes.

1. Model theory. 2. Algebra. I. Title.

II. Series: Lecture notes in mathematics (Berlin)

QA3.L28 no. 521 [QA9.7] 510'.8s [512'.02] 76-15388

---

AMS Subject Classifications (1970): 02H15

---

ISBN 3-540-07696-4 Springer-Verlag Berlin · Heidelberg · New York  
ISBN 0-387-07696-4 Springer-Verlag New York · Heidelberg · Berlin

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, reprinting, re-use of illustrations, broadcasting, reproduction by photocopying machine or similar means, and storage in data banks.

Under § 54 of the German Copyright Law where copies are made for other than private use, a fee is payable to the publisher, the amount of the fee to be determined by agreement with the publisher.

© by Springer-Verlag Berlin · Heidelberg 1976

Printed in Germany

Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr.

Table of Contents

Introduction .. .. .	1
O. Basic Model Theory .. .. .	4
§1. First order languages. First order sentences .. .. .	4
§2. n-types and saturation .. .. .	9
§3. Unions of Chains .. .. .	12
§4. The method of diagrams .. .. .	14
§5. Cardinal transfer theorems .. .. .	16
§6. Definable sets .. .. .	17
I. Transfer Theorems in Algebra .. .. .	21
§1. Polynomial Maps on $C^n$ .. .. .	21
§2. The Nullstellensatz and Hilbert's seventeenth problem ..	22
§3. Notes .. .. .	30
Exercises .. .. .	30
II. The Ax-Kochen-Ershov Transfer Principle: (Diophantine Problems over Local Fields) .. .. .	32
§1. Valued fields .. .. .	32
§2. p-adic fields .. .. .	36
§3. Complete fields and Hensel fields .. .. .	38
§4. Normalized cross sections .. .. .	44
§5. Artin's conjecture .. .. .	46
§6. Artin-Schreier theory for p-adic fields .. .. .	50
§7. Puiseux series fields .. .. .	61
§8. Notes .. .. .	64
Exercises .. .. .	65
III. Existentially Complete Structures .. .. .	67
§1. Existentially complete structures .. .. .	68
§2. Infinitely generic structures .. .. .	74
§3. Finitely generic structures .. .. .	83
§4. Existentially complete commutative rings .. .. .	92
§5. Rings without nilpotents .. .. .	95
§6. A generalized Nullstellensatz .. .. .	103
§7. Notes .. .. .	104
Exercises .. .. .	105
IV. Existentially Complete Division Rings .. .. .	108
§1. Amalgamating division rings .. .. .	109
§2. Existentially complete division rings - algebraic aspects	124
§3. Existentially complete division rings - model theoretic aspects .. .. .	132
§4. Existentially complete groups .. .. .	145

IV

§5. Notes .. .. .	149
Exercises .. .. .	150
V. Existentially Complete Modules .. .. .	156
§1. Z-modules .. .. .	156
§2. R-modules .. .. .	158
§3. Existence of model companions .. .. .	162
§4. Coherent rings .. .. .	167
§5. Existentially complete modules .. .. .	170
§6. Notes .. .. .	182
Exercises .. .. .	182
VI. Complete Theories of Abelian Groups .. .. .	185
§1. Structure of saturated abelian groups .. .. .	186
§2. Construction of saturated groups .. .. .	197
§3. Notes .. .. .	204
Exercises .. .. .	205
VII. $\aleph_1$ -categorical Fields .. .. .	207
§1. The Nullstellensatz revisited .. .. .	208
§2. Introduction to categoricity .. .. .	210
§3. $\aleph_0$ -categorical fields .. .. .	214
§4. Notes .. .. .	221
Exercises .. .. .	221
Bibliography .. .. .	223
Subject Index .. .. .	227
Index of Principal Notation .. .. .	233