Contents

Introduction **1**

Chapter 1. Formulation of Inverse Logarithmic Potential Problem. Fundamental Equation **7**

§1.1 Formulation of inverse problem 7
§1.2 Nonlinear boundary value problem for mapping function 13
§1.3 The analytic continuation of the potential across a boundary 16
§1.4 The boundary analyticity of domain is a solution to an inverse problem . . 23
§1.5 The structure of inverse problem solution. final solvability. Examples . . . 28

Chapter 2. Local Solvability of an Inverse Problem **41**

§2.1 Univalent function variation 41
§2.2 Local theorem of existence 42
§2.3 Linearization of the boundary value problem 44
§2.4 The auxiliary problem 46
§2.5 The Newton-Kantorovitch method. 47
§2.6 The explicit solution of the linear problem. 49
§2.7 The local uniqueness theorem. 49
§2.8 The density variation. The equivalent solution set of the inverse problems. Remarks. 50
§2.9 The complex-valued density case. 52
§2.10 Existence theorems for the inverse problem for small constant densities . . 56
§2.11 Proof of theorems 58

Chapter 3. The Estimate of Bounded Univalent Function Coefficients and Univalent Polynomials **71**

§3.1 Classical estimates. Classes of bounded functions and with bounded image area. 71
§3.2 The estimate of univalent polynomials coefficients. 74
§3.3 The Diedonne-Horowitz inequalities for univalent polynomials 77
§3.4 Numerical estimates of univalent polynomials coefficients 81

Chapter 4. Mass Potential Estimates. Necessary Conditions for Solvability. A Priori Estimates for Inverse Problem Solution **85**

Materiale protetto da copyright

§4.1 Exact estimates for a mass potential gradient in the three-dimensional case. Extremal domain 85
§4.2 Exact estimates of logarithmic mass potential 91
§4.3 *A priori* estimates for inverse potential problem solution 93
§4.4 On zeros of a potential mass gradient 97
§4.5 Estimates of mass potential derivatives in a fixed angle. 100
§4.6 The estimates of the mass potential derivatives in the disk 103
§4.7 The estimate of the mass potential based on the Calderon-Zygmund results for the singular integral 109
§4.8 The necessary solvability conditions, *a priori* estimates — using the univalent function theory 113

Chapter 5. The Continuation by the Parameter of an Inverse Problem Solution 125
§5.1 The dependence of an inverse problem solution on the parameter – a constant density 125
§5.2 The theorem on the continuation of a solution by the parameter 129
§5.3 Inverse potential problems and univalent functions 132

Chapter 6. On the Analyticity and Smoothness of an Inverse Problem Solution 139
§6.1 Theorem on the smoothness of inverse problem solutions 140
§6.2 Applications of the theorem on smoothness, in connection with free boundary smoothness 148
§6.3 Analytical continuation of the potential through the angle points 150

Chapter 7. Inverse Linear Problem. Determination of a Density of the Given Domain by its Exterior Potential 155
§7.1 Existence theorems. The particular solutions of the inhomogeneous problem construction 155
§7.2 Solution of the homogeneous problem. Density of the vanishing external potential 158
§7.3 Special classes of domains 162
§7.4 Determination of a body's density by the given potential of an elliptic equation 167
§7.5 Linear inverse problem in the classes L_p 170

Chapter 8. Conjugation of Harmonic and Analytic Functions: Direct and Inverse Problems. 173
§8.1 Problems of the linear conjugation for harmonic and analytic functions. . . 174
§8.2 The particular cases, modifications, applications and generalization of the base conjugation problem. 183
§8.3 Formulation of inverse problems. 189
§8.4 Applied inverse problems. 193
§8.5 Interior inverse problems. 198

Materiale protetto da copyright

Chapter 9. Applications in Gravity Prospecting and in Magnetic Prospecting **205**

§9.1 Setting of problems. The algorithm for numerical construction of the equivalent solutions set by the analytically given field 206

§9.2 On one approximation method (analytic continuation) for gravitational fields 209

§9.3 Examples of the numerical constructing of the equivalent solutions family. 210

§9.4 Approximation of the anomaly field and determining the object by the random search method 219

Bibliography **229**

Materiale protetto da copyright