American Mathematical Society

Colloquium Publications
Volume 49

Attractors for Equations of Mathematical Physics

Vladimir V. Chepyzhov Mark I. Vishik



American Mathematical Society Providence, Rhode Island

Contents

Introduction		1
Part 1	1. Attractors of Autonomous Equations	15
Chapte	er I. Attractors of Autonomous Ordinary Differential Equations	17
_	Semigroups and attractors	17
	Examples of ordinary differential equations and their attractors	21
Chapte	er II. Attractors of Autonomous Partial Differential Equations	27
1.	Function spaces and embedding theorems	28
2.	Operator semigroups. Basic notions	35
3.	Attractors of semigroups	37
4.	Reaction-diffusion systems	38
5.	2D Navier–Stokes system	46
6.	Hyperbolic equation with dissipation	49
Chapte	er III. Dimension of Attractors	51
1.	Fractal and Hausdorff dimension	51
2.	Dimension of invariant sets	53
3.	Optimization of the bound for the fractal dimension	59
4.	Application to semigroups	62
5.	Applications to evolution equations	65
6.	Lower bounds for the dimension of attractors	73
Part 2	2. Attractors of Non-autonomous Equations	77
Chapte	er IV. Processes and Attractors	79
1.	Symbols of non-autonomous equations	80
2.	Cauchy problem and processes	82
3.	Uniform attractors	83
4.	Haraux's example	85
5.	The reduction to a semigroup	86
6.	On uniform (w.r.t. $\tau \in \mathbb{R}$) attractors	92
Chapte	Chapter V. Translation Compact Functions	
1.	Almost periodic functions	95
2.	Translation compact functions in $C(\mathbb{R}; \mathcal{M})$	97
3.	Translation compact functions in $L_p^{loc}(\mathbb{R}; \mathcal{E})$ Translation compact functions in $L_{p,w}^{loc}(\mathbb{R}; \mathcal{E})$	101
4.	Translation compact functions in $L_{n,w}^{loc}(\mathbb{R};\mathcal{E})$	104
5	Other translation compact functions	106

x CONTENTS

Chapter VI. Attractors of Non-autonomous Partial Differential Equations	107
1. 2D Navier–Stokes system	107
2. Non-autonomous reaction-diffusion systems	114
3. Non-autonomous Ginzburg-Landau equation and others	118
4. Non-autonomous damped hyperbolic equations	119
Chapter VII. Semiprocesses and Attractors	129
1. Families of semiprocesses and their attractors	129
2. On the reduction to the semigroup	132
3. Non-autonomous equations with tr.c. on \mathbb{R}_+ symbols	135
4. Prolongations of semiprocesses to processes	137
5. Asymptotically almost periodic functions	140
6. Non-autonomous equations with a.a.p. symbols	143
7. Cascade systems and their attractors	146
Chapter VIII. Kernels of Processes	149
1. Properties of kernels	149
2. On the dimension of connected sets	153
3. Dimension estimates for kernel sections	155
4. Applications to non-autonomous equations	157
Chapter IX. Kolmogorov ε -Entropy of Attractors	163
1. Estimates of the ε -entropy	163
2. Fractal dimension of attractors	173
3. Functional dimension and metric order	176
4. Applications to evolution equations	177
5. η -entropy and metric order of Σ	188
6. ε -entropy in the extended phase space	192
Part 3. Trajectory Attractors	197
Chapter X. Trajectory Attractors of Autonomous Ordinary Differential	
Equations	199
1. Preliminary propositions	200
2. Construction of the trajectory attractor	203
3. Examples of equations	205
4. Dependence on a parameter	207
Chapter XI. Attractors in Hausdorff Spaces	211
1. Some topological preliminaries	211
2. Semigroups in topological spaces and attractors	214
3. Applications to $(\mathcal{M}, \mathfrak{T})$ -attractors	218
Chapter XII. Trajectory Attractors of Autonomous Equations	219
1. Trajectory spaces of evolution equations	219
2. Existence of trajectory attractors	222
3. Trajectory and global attractors	224
Chapter XIII. Trajectory Attractors of Autonomous Partial Differential	
Equations	229
1. Autonomous Navier–Stokes systems	229

CONTENTS xi

2. Autonomous hyperbolic equations	242
3. Hyperbolic equations depending on a parameter	251
Chapter XIV. Trajectory Attractors of Non-autonomous Equations	
1. Non-autonomous equations, their symbols, and trajectory spaces	260
2. Existence of uniform trajectory attractors	262
3. Equations with symbols on the semiaxis	266
Chapter XV. Trajectory Attractors of Non-autonomous Partial	
Differential Equations	269
1. Non-autonomous Navier-Stokes systems	269
2. Trajectory attractor for 2D Navier–Stokes system	278
3. Reaction-diffusion systems	282
4. Non-autonomous hyperbolic equations	292
Chapter XVI. Approximation of Trajectory Attractors	299
1. Trajectory attractors of non-autonomous ordinary	
differential equations	299
2. Trajectory attractors of Galerkin systems	302
3. Convergence of trajectory attractors of Galerkin systems	303
Chapter XVII. Perturbation of Trajectory Attractors	305
1. Trajectory attractors of perturbed equations	305
2. Dependence of trajectory attractors on a small parameter	307
Chapter XVIII. Averaging of Attractors of Evolution Equations with	
Rapidly Oscillating Terms	311
1. Averaging of rapidly oscillating functions	311
2. Averaging of equations and systems	320
3. Perturbation with rapidly oscillating terms	341
Appendix A. Proofs of Theorems II.1.4 and II.1.5	
Appendix B. Lattices and Coverings	
Bibliography	
Index	