Graduate Texts in Mathematics 208

Springer Science+Business Media, LLC

Graduate Texts in Mathematics

(continued after index)

Ward Cheney

Analysis for Applied Mathematics

With 27 Illustrations

Springer

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Mathematics Subject Classification (2000): 46Bxx, 65L60, 32Wxx, 42B10

Library of Congress Cataloging-in-Publication Data
Cheney, E. W. (Elliott Ward), 1929–
Analysis for applied mathematics / Ward Cheney.
p. cm. — (Graduate texts in mathematics ; 208)
Includes bibliographical references and index.

ISBN 978-1-4419-2935-8 ISBN 978-1-4757-3559-8 (eBook)
DOI 10.1007/978-1-4757-3559-8

1. Mathematical analysis. I. Title. II. Series.
QA300.C4437 2001
515—dc21 2001-1020440

Printed on acid-free paper.

Production managed by Terry Kornak; manufacturing supervised by Jerome Basma.
Photocomposed from the author's TeX files.

9 8 7 6 5 4 3 2 1

SPIN 10833405

Preface

This book evolved from a course at our university for beginning graduate students in mathematics—particularly students who intended to specialize in applied mathematics. The content of the course made it attractive to other mathematics students and to graduate students from other disciplines such as engineering, physics, and computer science. Since the course was designed for two semesters duration, many topics could be included and dealt with in detail. Chapters 1 through 6 reflect roughly the actual nature of the course, as it was taught over a number of years. The content of the course was dictated by a syllabus governing our preliminary Ph.D. examinations in the subject of applied mathematics. That syllabus, in turn, expressed a consensus of the faculty members involved in the applied mathematics program within our department. The text in its present manifestation is my interpretation of that syllabus: my colleagues are blameless for whatever flaws are present and for any inadvertent deviations from the syllabus.

The book contains two additional chapters having important material not included in the course: Chapter 8, on measure and integration, is for the benefit of readers who want a concise presentation of that subject, and Chapter 7 contains some topics closely allied, but peripheral, to the principal thrust of the course.

This arrangement of the material deserves some explanation. The ordering of chapters reflects our expectation of our students: If they are unacquainted with Lebesgue integration (for example), they can nevertheless understand the examples of Chapter 1 on a superficial level, and at the same time, they can begin to remedy any deficiencies in their knowledge by a little private study of Chapter 8. Similar remarks apply to other situations, such as where some point-set topology is involved; Section 7.6 will be helpful here. To summarize: We encourage students to wade boldly into the course, starting with Chapter 1, and, where necessary, fill in any gaps in their prior preparation. One advantage of this strategy is that they will see the *necessity* for topology, measure theory, and other topics — thus becoming better motivated to study them. In keeping with this philosophy, I have not hesitated to make forward references in some proofs to material coming later in the book. For example, the Banach contraction mapping theorem is needed at least once prior to the section in Chapter 4 where it is dealt with at length.

Each of the book's six main topics could certainly be the subject of a year's course (or a lifetime of study), and many of our students indeed study functional analysis and other topics of the book in separate courses. Most of them eventually or simultaneously take a year-long course in analysis that includes complex analysis and the theory of measure and integration. However, the applied mathematics course is typically taken in the first year of graduate study. It seems to bridge the gap between the undergraduate and graduate curricula in a way that has been found helpful by many students. In particular, the course and the book certainly do not presuppose a thorough knowledge of integration theory nor of topology. In our applied mathematics course, students usually enhance and reinforce their knowledge of undergraduate mathematics, especially differential equations, linear algebra, and general mathematical analysis. Students may, for the first time, perceive these branches of mathematics as being essential to the foundations of applied mathematics.

The book could just as well have been titled *Prolegomena to Applied Mathematics*, inasmuch as it is *not* about applied mathematics itself but rather about topics in analysis that impinge on applied mathematics. Of course, there is no end to the list of topics that could lay claim to inclusion in such a book. Who is bold enough to predict what branches of mathematics will be useful in applications over the next decade? A look at the past would certainly justify my favorite algorithm for creating an applied mathematician: Start with a pure mathematician, and turn him or her loose on real-world problems.

As in some other books I have been involved with, I owe a great debt of gratitude to Ms. Margaret Combs, our departmental TEX-pert. She typeset and kept up-to-date the notes for the course over many years, and her resourcefulness made my burden much lighter.

The staff of Springer–Verlag has been most helpful in seeing this book to completion. In particular, I worked closely with Dr. Ina Lindemann and Ms. Terry Kornak on editorial matters, and I thank them for their efforts on my behalf. I am indebted to David Kramer for his meticulous copy-editing of the manuscript; it proved to be very helpful in the final editorial process.

I thank my wife, Victoria, for her patience and assistance during the period of work on the book, especially the editorial phase. I dedicate the book to her in appreciation.

I will be pleased to hear from readers having questions or suggestions for improvements in the book. For this purpose, electronic mail is efficient: `cheney@math.utexas.edu`. I will also maintain a web site for material related to the book at `http://www.math.utexas.edu/users/cheney/AAMbook`

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