## Translations of

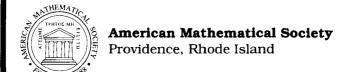
## MATHEMATICAL MONOGRAPHS

Volume 174

## Second Order Elliptic Equations and Elliptic Systems

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