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Bruce Chandler
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The History of Combinatorial
Group Theory:
A Case Study in the History
of Ideas



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Preface

One of the pervasive phenomena in the history of science is the development of independent disciplines from the solution or attempted solutions of problems in other areas of science. In the Twentieth Century, the creation of specialties within the sciences has accelerated to the point where a large number of scientists in any major branch of science cannot understand the work of a colleague in another subdiscipline of his own science. Despite this fragmentation, the development of techniques or solutions of problems in one area very often contribute fundamentally to solutions of problems in a seemingly unrelated field. Therefore, an examination of this phenomenon of the formation of independent disciplines within the sciences would contribute to the understanding of their evolution in modern times.

We believe that in this context the history of combinatorial group theory in the late Nineteenth Century and the Twentieth Century can be used effectively as a case study. It is a reasonably well-defined independent specialty, and yet it is closely related to other mathematical disciplines. The fact that combinatorial group theory has, so far, not been influenced by the practical needs of science and technology makes it possible for us to use combinatorial group theory to exhibit the role of the intellectual aspects of the development of mathematics in a clearcut manner. There are other features of combinatorial group theory which appear to make it a reasonable choice as the object of a historical study. It is a rather young discipline, being approximately a century old. The literature, although not small (comprising about 5000 papers) was surveyed in 1939 by WILHELM MAGNUS and in 1974 by GILBERT BAUMSLAG. Nearly the entire body of research in the field is due to mathematicians who either are still alive or who were the teachers or senior colleagues of living mathematicians. This makes it possible to supplement the written tradition with oral information which is particularly valuable when dealing with questions of motivation for a particular investigation or of the transfer of ideas.

We have supplemented the mathematical discussions with some biographical data and with general descriptions of the external conditions for mathematical research, using examples from our special field as illustrations. In Chapter II.14 we also try to describe some of the effects of the rapid growth of mathematical research.

We gratefully acknowledge the help which we received from many sources in writing this book. We cannot enumerate the names of the many individuals who helped us with information and advice. Apart from these, we wish to thank the National Science Foundation for its generous support which covered all of Part I and without which we would not have begun our project. The hospitality of the Mathematics Research center of Warwick University at Coventry, England enabled us to interview a great many group theorists who visited there in 1978. The Polytechnic Institute of New York, through its Department of Mathematics, provided us with much of the technical and bibliographic help which we needed. And the excellent and well-organized library of the Courant Institute of Mathematical Sciences at New York University reduced to a pleasant minimum the normally onerous task of getting hold of the documentary sources for a historical study.

November 1982

BRUCE CHANDLER
WILHELM MAGNUS

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